1) (40 pts, 2 pages) Consider the point $P(1, 4, -1)$, $Q(2, 4, 3)$ and $R(1, 5, -4)$ in $\mathbb{R}^3$.

a) Find the equation of the line $L_1$ through $P$ and $Q$. Express your answer both in vector form and in symmetric form.

b) Find the equation of the line $L_2$ through $P$ and $R$. Express your answer both in vector form and in symmetric form.

c) Find a vector perpendicular to both $L_1$ and $L_2$.

d) Find the equation of a plane containing $P$, $Q$, and $R$.

e) Find the equation of a line through $P$ that is perpendicular to both $L_1$ and $L_2$. (You only have to give one form of the equation of this line — your pick which one.)

2. (30 points) Consider the parametrized curve $r(t) = \langle t - 4, t^2 + 5, \frac{2t^3 + 1}{3} \rangle$, where we think of $t$ as time.

a) Find the velocity and speed as a function of $t$.

b) Find a unit vector tangent to the curve at the point $\langle -3, 6, 1 \rangle$.

c) Find the arc-length of the portion of the curve that runs from $t = 1$ to $t = 3$.

3. (30 pts, 2 pages)

Consider the function $f(x, y) = e^{x-1} \left[ 1 + \ln(x + 3xy) \right]$.

a) Find the partial derivatives $f_x$, $f_y$, $f_{xx}$, $f_{xy}$, and $f_{yy}$ as functions of $x$ and $y$.

b) Find the equation of the plane tangent to the surface $z = f(x, y)$ at $(1, 0, 1)$.

c) Use first derivatives to estimate the value of $f(0.99, 0.02)$. Extra credit (5 points): Use second derivatives to get a better estimate of $f(0.99, 0.02)$. (Yes, I expect you to do the arithmetic by hand)