M408N First Midterm Exam, September 24, 2014

1) Let $Q(t)$ be the percentage of students at UT who have dropped a class in week t, and suppose that the rate equation for Q is

$$Q' = 0.2Q - 0.005Q^2$$

and that $Q(5) = 10$.

a) Use Euler's method with step size $h = 2$ to estimate $Q(7)$.

b) Use Euler's method with step size $h = 2$ to estimate $Q(3)$.

c) Use Euler's method with step size $h = 1$ to estimate $Q(7)$.

2) Here is a table of values of the function $f(x) = \tan(x \text{ degrees})$.

x (in degrees)	$tan(x)$
44	0.9656887748
44.9	0.99651541969
44.99	0.99965099505
45	1
45.01	1.00034912679
45.1	1.00349676506
46	1.03553031379

a) Find $f'(45)$ to at least 4 decimal places. [Note: you may have already learned a formula for the derivative of the tangent function, but that formula probably uses radians rather than degrees, so it gives the wrong answer.]

b) Use this information and the microscope equation to estimate $f(50)$.

3) Suppose that $f(x)$ is a differentiable function with $f(2) = -3$ and $f'(2) = 7$.

a) Find an equation for the line tangent to $y = f(x)$ at $(2, -3)$.

b) Estimate the values of $f(2.05)$ and $f(1.9)$.

4) The following model is NOT the SIR model, but it uses the same sort of reasoning as the SIR model. It's up to you to provide the details.

A hospital is treating patients who have a particular non-fatal disease. On average, patients spend 8 days in the hospital before being released. Let $P(t)$ be the number of patients in the hospital at time t, and let $R(t)$ be the number of patients who have been released. Every day, 20 new patients are admitted to the hospital.

a) Write down a set of rate equations for P and R. **Explain your reasoning!!** What does each term in the rate equation represent?

b) If there are 100 patients in the hospital at a particular time, is the number of patients increasing or decreasing? What if there are 200 patients?

5) a) Find the derivative of the function $f(t) = t^3 - 3t + 2$. You may use the formulas from Section 3.5.

b) The position of a particle is given by $x(t) = t^3 - 3t + 2$, where t is measured in seconds and x is measured in feet. How fast is the particle moving at time $t = 2$? Is it moving forwards or backwards?

c) At what time(s) is the particle's velocity equal to zero?