Last week the European Space Agency landed a lander, called Philae, on a comet 300 million miles from Earth. The trouble is that the lander had limited battery power, and a lot of scientific experiments to do. Philae made it through the primary to-do list before its battery went dead.

Suppose that Philae’s last experiment requires 500 units of energy. However, the battery will only be capable of putting out $300e^{-t/2}$ units of energy per hour at a time $t$ hours from now. Will Philae be able to complete its last task? If so, how long will it take?

That’s a lot to think about, so let’s break it down into steps.

a) How much energy does the battery put out in a time interval of length $\Delta t$?

*Putting out energy at rate $300e^{-t/2}$ for time $\Delta t$ means putting out $300e^{-t/2}\Delta t$ units of energy.*

b) Write down a Riemann sum that approximates the energy output between time $a$ and time $b$.

$$\sum_{i=1}^{N} 300e^{-t_i/2} \Delta t,$$

where $\Delta t = (b - a)/N$, $t_i = a + i\Delta t$, and $t_i^*$ is a sample point chosen somewhere between $t_{i-1}$ and $t_i$.

c) By taking a limit as the number of time intervals goes to infinity, write down an integral that gives the energy output exactly.

*The limit of this sum, by definition, is the integral*

$$\int_{a}^{b} 300e^{-t/2} dt.$$

*The name of the variable is irrelevant. We could just as well write $\int_{a}^{b} 300e^{-s/2} ds$.*

d) Now let $A(t)$ be the energy output between time 0 and time $t$. Find a formula for $A(t)$. 

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Just plug in \( b = t \) and \( a = 0 \) to get \( \int_0^t 300e^{-s/2}ds \). We do this by u-substitution with \( u = -s/2 \), hence \( du = -ds/2 \) and \( ds = -2du \). Our integral becomes \( \int -600e^u du = -600e^u \) (plus a constant, which we can take to be zero). Converting back to \( s \) we have \(-600e^{-s/2}\). Plugging in \( s = t \) and \( s = 0 \) then gives

\[
A(t) = 600 - 600e^{-t/2}.
\]

e) Can you solve \( A(t) = 500 \)? If so, what is \( t \)?

\[
\begin{align*}
500 &= 600 - 600e^{-t/2} \\
-100 &= -600e^{-t/2} \\
1/6 &= e^{-t/2} \\
\ln(1/6) &= -t/2 \\
t &= -2\ln(1/6) = 2\ln(6) \approx 3.5835
\end{align*}
\]

Since the problem has a solution, Philae DOES have enough battery power left to finish its job (as actually happened), and to do it in a little over 3.5 hours.