M408R in-class problem, November 19, 2014

Last week the European Space Agency landed a lander, called Philae, on a comet 300 million miles from Earth. The trouble is that the lander had limited battery power, and a lot of scientific experiments to do. Philae made it through the primary to-do list before its battery went dead.

Suppose that Philae's last experiment requires 500 units of energy. However, the battery will only be capable of putting out $300e^{-t/2}$ units of energy per hour at a time t hours from now. Will Philae be able to complete its last task? If so, how long will it take?

That's a lot to think about, so let's break it down into steps.

a) How much energy does the battery put out in a time interval of length Δt?

Putting out energy at rate $300e^{-t/2}$ *for time* Δt *means putting out* $300e^{-t/2}\Delta t$ *units of energy.*

b) Write down a Riemann sum that approximates the energy output between time a and time b.

$$\sum_{i=1}^{N} 300e^{-t_i^*/2}\Delta t,$$

where $\Delta t = (b-a)/N$, $t_i = a + i\Delta t$, *and* t_i^* *is a sample point chosen somewhere between* t_{i-1} *and* t_i.

c) By taking a limit as the number of time intervals goes to infinity, write down an integral that gives the energy output exactly.

The limit of this sum, by definition, is the integral

$$\int_a^b 300e^{-t/2}dt.$$

The name of the variable is irrelevant. We could just as well write $\int_a^b 300e^{-s/2}ds$.

d) Now let $A(t)$ be the energy output between time 0 and time t. Find a formula for $A(t)$.

Just plug in $b = t$ and $a = 0$ to get $\int_0^t 300e^{-s/2}ds$. We do this by u-substitution with $u = -s/2$, hence $du = -ds/2$ and $ds = -2du$. Our integral becomes $\int -600e^u du = -600e^u$ (plus a constant, which we can take to be zero). Converting back to s we have $-600e^{-s/2}$. Plugging in $s = t$ and $s = 0$ then gives

$$A(t) = 600 - 600e^{-t/2}.$$

e) Can you solve $A(t) = 500$? If so, what is t?

$$\begin{aligned}
500 &= 600 - 600e^{-t/2} \\
-100 &= -600e^{-t/2} \\
1/6 &= e^{-t/2} \\
\ln(1/6) &= -t/2 \\
t &= -2\ln(1/6) = 2\ln(6) \approx 3.5835
\end{aligned}$$

Since the problem has a solution, Philae DOES have enough battery power left to finish its job (as actually happened), and to do it in a little over 3.5 hours.