M408M Third Midterm Exam, November 21, 2013

1a) Compute the gradient of the function $f(x, y, z) = xe^y + yz^2 + xz$.

b) Let S be the surface $xe^y + yz^2 + xz = 3$. Compute the equation of the plane tangent to S at the point $P(1, 0, 2)$.

c) Staying on the surface S, estimate the value of y when $x = 1.01$ and $z = 2.02$.

2. I want to build a storage shed in my back yard with a square footprint. That's four walls (all of the same size) and a square roof (no floor). The materials for the walls cost \$1 per square foot, while the materials for the roof cost \$4 per square foot. What are the dimensions of the shed of maximum volume that I can build for \$108?

3. For each of the following functions, find the critical point(s) and determine which are maxima, which are minima, which are saddles, and which are something else.

a) $f(x, y) = xy - x - y - x^2 - y^2$

b) $f(x, y) = e^{x^2-y^2}$.

4. a) Compute the iterated integral

$$\int_0^2 \int_0^x 6ye^{(x^3)} dydx.$$

b) We want to compute the double integral $\iint_R f(x, y)dA$, where $f(x, y) = \ln(e^{xy} + 7)$ and R is the region between the curve $y = e^x$ and the line $y = 1 + \frac{e^2-1}{2}x$. Express this double integral as an **iterated integral** where we integrate first over y and then over x. Be clear about what the limits of integation are for both the x-integral and the y-integral. (For heavens sake, **do not attempt** to evaluate the iterated integral. It's a mess.)

c) Now set up (but do not evaluate!) another iterated integral that computes $\iint_R f(x, y)dA$ where now we integrate first over x and then over y.