M408N Class summary

1. Close is good enough! (limits)

   (a) Meaning of limit statements

   i. Limits as $x \to a$, $x \to a^+$, $x \to a^-$
   ii. Limits as $x \to \infty$ or $x \to -\infty$
   iii. Limits can equal L, $+\infty$, $-\infty$
   iv. Meaning of ∞ – a process, not a number

   (b) Using limit laws

   i. Sum, product, and quotient rules
   ii. Squeeze (sandwich) theorem

   (c) Continuity

   i. Definition: $\lim_{x \to a} f(x) = f(a)$.
   ii. Identifying when a function is continuous
   iii. Intermediate value theorem.

2. Track the changes (derivatives)

   (a) Definitions as a limit:

   i. $f'(a) = \lim_{x \to a}[f(x) - f(a)]/(x - a)$
   ii. $f'(a) = \lim_{h \to 0}[f(a + h) - f(a)]/h$.
   iii. $f'(x) = \lim_{h \to 0}[f(x + h) - f(x)]/h$.

   (b) Interpretation as:

   i. Instantaneous rate of change
   ii. Slope of tangent line
   iii. Best linear approximation (differentials and linearization)
   iv. Conversion factor from dx to dy.

   (c) How to compute it:

   i. Building blocks: derivatives of x^n, e^x, trig functions and logs.
   ii. Sum, product and quotient rules
   iii. Chain rule
   iv. Implicit differentiation and derivatives of inverse functions.
   v. Logarithmic differentiation

(d) What's it good for? (Besides max/min)

i. Understanding shapes of graphs
ii. Related rates
iii. L'Hôpital's rule
iv. Linearization/differentials
v. Solving equations with Newton's method
vi. Higher derivatives give curvature, etc.
vii. Velocity, acceleration, marginal quantities in economics
viii. Differential equations and anti-derivatives

3. What goes up has to stop before coming down (max/min)

(a) Absolute maxima and minima

i. Extreme value theorem. Every continuous function on a closed interval has a maximum/minimum.
ii. Max/min occur either at
A. Endpoints, or
B. Critical points, where $f'(x) = 0$ or doesn't exist.

(b) Local maxima and minima

i. First derivative test: Is the function increasing, decreasing, or flat?
ii. 2nd derivative test: Is a critical point a local max ($f'' < 0$) , local min($f'' > 0$), or unclear ($f'' = 0$ or f'' DNE).

(c) Using first and second derivatives to understand a function.

i. Sign charts for f, f', f''.
ii. Understanding what signs of f' and f'' tell you.

(d) Optimization

i. Draw a picture
ii. Define your variables, and figure out what you are trying to maximize or minimize
iii. Write down relations between variables. Eliminate all but one variable.
iv. Find critical points and compare values.
v. Interpret!

4. The whole is the sum of the parts. (integration)
   (a) The idea: Bulk quantities are limits of sums.
   (b) Estimating with rectangles – Riemann sums
   (c) Definite integrals are limits of Riemann sums as you slice finer and finer ("Close is good enough" strikes again)
5. The whole change is the sum of the partial changes (FTC)
   (a) Keeps straight the difference between definite integals, indefinite integrals and anti-derivatives.
   (b) Getting definite integrals by anti-differentiation: $\int_a^b f(x)dx = F(b) - F(a)$.
   (c) Indefinite integrals are anti-derivatives: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$.
   (d) Combine with chain rule to compute $\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)dt$.