M408N Second Midterm Exam, October 25, 2012

1. (42 points) Compute dy/dx in each of these situations. You do not need to simplify:

a) $y = \tan^{-1}(2x)$

b) $y = \frac{\ln(x)+5}{\sin(x)+2}$

c) $y = \ln(xe^x + 1)$

d) $y = \sin(x)\sin^{-1}(x)$

e) $y = x^{3x}$

f) $y = e^{\sin(\cos(\ln(x)))}$

g) $xe^y = \ln(xy + 1) + 8$

2) (12 pts) Derivatives and limits

a) Let $f(x)$ be a function that is differentiable at $x = a$. State the definition of $f'(a)$ as a limit. [There are two standard forms – either one will do.]

b) Compute $\lim\limits_{h\to 0} \dfrac{\sin(\pi(1+h)^2) - \sin(\pi)}{h}$

c) Compute $\lim\limits_{x\to 5} \dfrac{x^2e^x - 25e^5}{x-5}$

3. (12 points) Consider the function $f(x) = e^{\sin(3x)}$.

a) Find the equation of the line tangent to the curve $y = f(x)$ at $x = 0$.

b) Find the linearization of the function $f(x) = e^{\sin(3x)}$ around $a = 0$.

c) Using either differentials of linear approximations, approximate the value of $e^{\sin(0.3)}$.

4. (12 pts) Evaluate the following trig expressions:

a) $\cos(4\pi/3)$

b) $\tan(3\pi/4)$

c) $\sin^{-1}(\sin(5\pi/6))$

5. (12 pts) Related rates. A team is digging a hemispherical hole. They start by making a small hole, and enlarge it in all directions so that it always has the shape of a hemisphere. [Note: the volume of a sphere is $\frac{4}{3}\pi r^3$, where r is the radius.] They are excavating dirt at a rate of 40 cubic feet per minute. We want to understand how fast the radius of the hole is increasing.

a) Draw a picture of the situation. Label your variables. Write down an equation relating the variables.

b) At the instant that the radius reaches 10 feet, how fast is the radius increasing?

6. (10 points) Sketch the graph of the derivative of the function whose graph is shown below. I don't expect you to get the details of the derivative right, but you should clearly show where the derivative is positive, negative and zero, where it is big and where it is small. Draw your graph directly under the one shown, so that the x-values can be compared.