M408N Third Midterm Exam, November 21, 2016

1. (2 pages, 34 points) Derivatives and curves. Consider the function $f(x) = x^4 - 4x^3 + 4x^2$.

a) Make a sign chart for f. Where is $f(x)$ positive? Negative? Zero?

b) Make a sign chart for f'. What are the critical points? Identify which critical points are local maxima, which are local minima, and which are neither.

c) Make a sign chart for f''. [Note: this is the only place in the problem where numbers that aren't integers might appear.]

d) Sketch the curve $y = f(x)$. It doesn't have to be precise, but it should be positive/negative in the right places, should be increasing/decreasing in the right places, should be concave up/down in the right places, and should have the correct asymptotes (if any).

2. (2 pages, 36 points) NASA is designing a rectangular solar panel for a mission to Mars. Because of the bizarre (and totally made up) constraints of space travel, a vertical strut of height y meters requires e^y kg of material, while a horizontal strut of length x meters requires x kg of material. A panel includes two vertical struts (left and right) and two horizontal struts (top and bottom), and we can only bring 12 kg of building material. What are the dimensions of the panel that will maximize the area? We'll tackle this step by step.

a) Draw a picture. Let x be the width of the panel and y the height.

b) What is the relation between x and y? [Remember that we are using a total of 12kg of building material]

c) Eliminate x, and express the area in terms of y only.

d) Take the derivative with respect to y and set it equal to zero to get an equation for y.

e) This equation cannot be solved exactly! Instead, get an approximate solution by starting with an initial guess $y = 1$, and running **one** iteration of Newton's method.

3. (2 pages, 30 points) Indeterminate forms and antiderivatives.

a) Compute $\lim_{x \to 3} \dfrac{x^3 - 27}{x^2 - 9}$

b) Compute $\lim_{x \to 0} \dfrac{\ln(\cos(x))}{\sin^2(x)}$

c) Compute $\lim_{x \to 0} (\cos(x))^{\csc^2(x)}$

d) Find the most general antiderivative of $f(x) = \cos(x) + e^{2x} + (x-1)^3$.

e) Find $f(x)$ if $f'(x) = \sec^2(x) + \frac{1}{(x+1)^2}$ and $f(0) = 5$.