1) Let \( Q(t) \) be the percentage of students at UT who have dropped a class in week \( t \), and suppose that the rate equation for \( Q \) is
\[
Q' = 0.2Q - 0.005Q^2
\]
and that \( Q(5) = 10 \).

a) Use Euler’s method with step size \( h = 2 \) to estimate \( Q(7) \).

One step of Euler’s method is really just the microscope equation: \( Q'(5) = 0.2(10) - 0.005(10)^2 = 1.5 \), so \( Q(7) \approx Q(5) + 2Q'(5) = 10 + 3 = 13 \).

b) Use Euler’s method with step size \( h = 2 \) to estimate \( Q(3) \).

\( Q(3) \approx Q(5) - 2Q'(5) = 10 - 3 = 7 \).

c) Use Euler’s method with step size \( h = 1 \) to estimate \( Q(7) \).

\( Q(6) \approx Q(5) + 1Q'(5) = 10 + 1.5 = 11.5 \). We then compute \( Q'(6) \approx 0.2(11.5) - 0.005(11.5)^2 = 1.63875 \) and \( Q(7) \approx Q(6) + 1Q'(6) \approx 11.5 + 1.63875 = 13.13875 \).

2) Here is a table of values of the function \( f(x) = \tan(x \text{ degrees}) \).

<table>
<thead>
<tr>
<th>( x ) (in degrees)</th>
<th>( \tan(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>0.9656887748</td>
</tr>
<tr>
<td>44.9</td>
<td>0.99651541969</td>
</tr>
<tr>
<td>44.99</td>
<td>0.99965099505</td>
</tr>
<tr>
<td>45</td>
<td>1</td>
</tr>
<tr>
<td>45.01</td>
<td>1.00034912679</td>
</tr>
<tr>
<td>45.1</td>
<td>1.00349676506</td>
</tr>
<tr>
<td>46</td>
<td>1.03553031379</td>
</tr>
</tbody>
</table>

a) Find \( f'(45) \) to at least 4 decimal places. [Note: you may have already learned a formula for the derivative of the tangent function, but that formula probably uses radians rather than degrees, so it gives the wrong answer.]

If you use forward differences, the best estimate is
\[
f'(45) \approx \frac{f(45.01) - f(45)}{0.01} = \frac{0.00034912679}{0.01} = 0.034912679.
\]

Using backwards differences, we get
\[
f'(45) \approx \frac{f(45) - f(44.99)}{0.01} = \frac{0.000349004941}{0.01} = 0.0349004941.
\]
Using centered differences gives

\[ f'(45) \approx \frac{f(45.01) - f(44.99)}{0.02} = \frac{0.006981317290}{0.02} = 0.03490658645. \]

No matter how you do it, the answer to 4 decimal places is 0.0349.

b) Use this information and the microscope equation to estimate \( f(50) \).

By the microscope equation \( f(50) \approx f(45) + 5f'(45) = 1 + 5(0.0349) = 1.1745 \). [FWIW, the actual value of \( f(50) \) turns out to be 1.19175]

3) Suppose that \( f(x) \) is a differentiable function with \( f(2) = -3 \) and \( f'(2) = 7 \).

a) Find an equation for the line tangent to \( y = f(x) \) at \((2, -3)\).

Since the slope is 7, point-slope form says that \( y + 3 = 7(x - 2) \), or \( y = 7(x - 2) - 3 \).

b) Estimate the values of \( f(2.05) \) and \( f(1.9) \).

\[ f(2.05) \approx -3 + 7(0.05) = -2.65. \quad f(1.9) \approx -3 + 7(-0.1) = -3.7. \]

4) The following model is NOT the SIR model, but it uses the same sort of reasoning as the SIR model. It’s up to you to provide the details.

A hospital is treating patients who have a particular non-fatal disease. On average, patients spend 8 days in the hospital before being released. Let \( P(t) \) be the number of patients in the hospital at time \( t \), and let \( R(t) \) be the number of patients who have been released. Every day, 20 new patients are admitted to the hospital.

a) Write down a set of rate equations for \( P \) and \( R \). Explain your reasoning!! What does each term in the rate equation represent?

Since the disease runs for 8 days (on average), we expect 1/8 of the patients to recover on any given day. So \( R' = (1/8)P \). Meanwhile, \( P \) is increasing by 20 new patients and decreasing by \( P/8 \), for a total of:

\[ P' = 20 - (P/8); \quad R' = P/8. \]

b) If there are 100 patients in the hospital at a particular time, is the number of patients increasing or decreasing? What if there are 200 patients?

When \( P = 100 \), \( P' = 20 - 100/8 = 7.5 > 0 \), so the number of patients is increasing. When \( P = 200 \), \( P' = 20 - 200/8 = -5 \), so \( P \) is decreasing.
5) a) Find the derivative of the function \( f(t) = t^3 - 3t + 2 \). You may use the formulas from Section 3.5.

The derivative of \( t^3 \) is \( 3t^2 \), the derivative of \(-3t\) is \(-3\), and the derivative of 2 is 0, so the derivative of \( t^3 - 3t + 2 \) is \( 3t^2 - 3 = 3(t^2 - 1) \).

b) The position of a particle is given by \( x(t) = t^3 - 3t + 2 \), where \( t \) is measured in seconds and \( x \) is measured in feet. How fast is the particle moving at time \( t = 2 \)? Is it moving forwards or backwards?

When \( t = 2 \), \( x' = 3(2)^2 - 3 = 9 \), so the particle is moving forwards at a rate of 9 feet/second.

c) At what time(s) is the particle’s velocity equal to zero?

Setting \( x' = 0 \) gives \( t^2 - 1 = 0 \), hence \( t = \pm 1 \).