M408R Third Midterm Exam, November 21, 2014

1) Compute the following integrals, each worth 6 points. Note that they don't all use the same technique! Please indicate what technique you are using.

a) $\int \cos(z)(1-\sin^2(z))dz$.

b) $\int \frac{dw}{3w+1}$.

c) $\int_0^1 \frac{2.4}{t^2+1}dt$.

d) $\int x^2 + \frac{1}{x^2}dx$.

e) $\int_1^e \frac{\ln(r)}{r}dr$.

f) $\int (y^2+1)^2 dy$.

g) $\int_{-1}^{3} |x|dx$. [Hint: Break it up as the sum of two definite integrals.]

2) Using anti-derivatives. Let $a(t) = -9.8$.

a) Find an anti-derivative $v(t)$ of $a(t)$ such that $v(1) = 39.2$. (There is one and only one correct answer.)

b) Find an anti-derivative $z(t)$ of $v(t)$ such that $z(1) = 44.1$.

c) For what values of t is $z(t) = 0$?

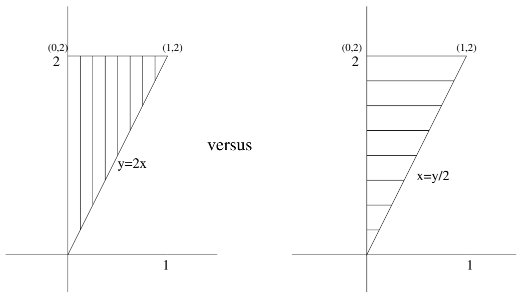
d) A cannonball is shot into the air. At time $t = 1$ second, it is 44.1 meters above the ground and is rising at 39.2 meters/second. At all times, it is accelerating *downwards* at $9.8m/s^2$ because of gravity. At what time will it hit the ground? At what time was it fired?

3) A snowstorm is expected to hit the town of Frostbite Falls tonight, starting at midnight. The rate at which snow falls is expected to be given by the function $r(t) = 5e^{-2t}$ inches/hour, where t is the number of hours since midnight. (E.g., 1:30 AM is $t = 1.5$, so at 1:30 AM the snow is coming down at a rate of $5e^{-3}$ inches/hour)

a) Write down a definite integral that gives the predicted total snowfall between midnight and 5AM. **Explain your reasoning!**

b) Now compute that integral. Give your answer first as an exact expression involving e, and then plug that expression into your calculator to get a numerical answer.

4) (2 pages!) Let T be a triangle in the x-y plane with vertices at the origin, at $(0, 2)$, and at $(1, 2)$. In other words, it is bounded by pieces of the y-axis, the line $y = 2$, and the line $y = 2x$ (aka $x = y/2$). We are going to compute the area of T in two different ways.



a) Write down an integral over x that gives the area of T. Explain where this integral comes from!

Your answer in (a) should encode what happens when you cut T up into a bunch of vertical strips, add up the approximate area of each strip, and take a limit (as in the first figure). Instead, imagine cutting T up into N **horizontal** strips, all of the same height Δy, as in the second figure.

b) What is the width of each strip, as a function of y?

c) What is the approximate area of each strip?

d) Write down a Riemann sum (based on this approach) that gives the approximate area of T.

e) Now write down an integral that is the limit of this Riemann sum as we slice things finer and finer. [Voila! You've found a second way to compute areas by integration.]