We aim to show that the contrapositive of a statement is equivalent to the original statement. In other words, that "P implies Q" is the same as "(not Q) implies (not P)". First we need some definitions and notation.

Let A be our universe. Let S_P denote the set of events in which P is true. We'll denote "not P" by $\sim P$, so $S_{\sim P} = A - S_P$ is the set of events in which P is false. Note that $A = S_P \cup S_{\sim P} = S_Q \cup S_{\sim Q}$. The phrase "P implies Q" is defined to mean $S_P \subset S_Q$. The empty set is denoted $\emptyset$.

Now for a lemma: "P implies Q" is equivalent to $S_P \cap S_{\sim Q} = \emptyset$.

The lemma is proved in two steps. First we show that $S_P \subset S_Q$ implies $S_P \cap S_{\sim Q} = \emptyset$, and then we show that $S_P \cap S_{\sim Q} = \emptyset$ implies $S_P \subset S_Q$.

Step 1: $S_P = S_P \cap A = S_P \cap (S_Q \cup S_{\sim Q}) = (S_P \cap S_Q) \cup (S_P \cap S_{\sim Q})$. The second term is disjoint from S_Q, so the only way it can be a subset of S_Q is if it is empty. Thus $S_P \cap S_q = \emptyset$.

Step 2: Suppose $S_P \cap S_q = \emptyset$. As before, we have that $S_P = (S_P \cap S_Q) \cup (S_P \cap S_{\sim Q})$. The first term is a subset of S_Q and the second term is empty, so $S_P \subset S_Q$. That completes the proof of the lemma.

Once we have the lemma, we note that "P implies Q" means $S_P \cap S_{\sim Q} = \emptyset$, while "(not Q) implies P", by the same lemma, means $S_{\sim Q} \cap S_{\sim\sim P} = \emptyset$. But $\sim\sim P = P$, so the conditions are the same.

That completes the formal proof. Let me restate this in informal terms. "P implies Q" means that Q is true whenever P is true. In other words, that you can't have P true and Q false. That's the lemma. Then, "(not Q) implies (not P)" means that you can't have (not Q) true and (not P) false. But that's the same thing!