The Genesis of Topology

In the beginning, God created the set, but the set was empty and void. And God said "Let there be elements, and let the elements combine into subsets, unions and intersections, and let there be formal logic." And God divided the true from the untrue, and saw that the true was good.

God then created operations on sets: functions and relations He created them. Among the functions were injections, surjections and bijections, and He saw that the bijective functions were good. Among the relations were equivalence, order, and partial order, and He saw that it was orderly.

God then created the integers and the real numbers. He used the least upper bound to divide the numbers above from the numbers below, and He used the integers to define the cardinality of finite sets. He divided the countable from the uncountable, and decreed the Axiom of Choice.

As a set, the world was complete, with subsets of various types and with the power set always ruling over the original set. And God rested, while the class took a midterm.

But the world lacked structure and continuity, so God said "Let there be topology," and there was topology. There were open sets satisfying three axioms, and Useful Properties of spaces defined in terms of open sets. And God said "Let there be bases with which to describe topologies, so that proofs boil down to working with basic open sets. Let ordered spaces and metric spaces have natural topologies, and let us define topologies (product and box) on arbitrary products of spaces. Let there be subspaces and quotient spaces, each with their own topologies, and let there be examples, like R_K and R_ℓ and the ordered square, to stretch our intuition."

And God said "Let closed sets be the complement of open sets. Let there be limits, such that closed sets contain their limit points. Let there be closures and interiors, and let there be Haussdorff spaces, where one-point sets are closed. Let there be continuous functions, where the preimage of open sets are open, and on metric spaces let the continuity also be described with ϵ and δ.

God was finished with his Creation, but it needed classification and study. So God created Man, to analyze his spaces and marvel at their goodness.

And Man divided spaces into irreducible pieces, pieces that could not be separated into disjoint nonempty open subsets. And he called those pieces "connected". He often visualized connectivity in terms of paths, but was careful to distinguish between connected components and path components,

since these were occasionally different. Man showed that the continuous image of a connected space was connected, and that the Intermediate Value Theorem holds on connected spaces.

Man then studied spaces large and small. The small, where every open cover has a finite subcover, he called Compact, and proved that functions on compact spaces are bounded and achieve their maxima. He also showed that sequences on compact sets always have convergent subsequences. On R^n, he showed that closed and bounded sets are compact.

Man then decided to classify spaces by countability and separation. He defined separation axioms T_0, T_1, T_2 (Haussdorff), T_3 (regular) and T_4 (normal), and showed that closed sets on normal spaces can be separated by functions, and used these functions to show that regular spaces with countable bases are metrizable.

Finally, he considered compactifications, and the ways that functions can be used to generate compactifications of spaces.

And then Man took a final exam, and rested. And Man saw that the rest was good.

Have a great summer!