M367K First Midterm Exam, February 7, 2008

1. Prove that every positive integer can be written as a sum of distinct powers of 2. (For instance, $13 = 1 + 4 + 8$.)

2. A real number is *algebraic* if it is a root of a polynomial with integer coefficients. For instance, $5/3$ is a root of $3x - 5$ and $\sqrt{2}$ is a root of $x^2 - 2$. Show that the set of algebraic real numbers is countable. [Hint: First show that the set of polynomials with integer coefficients in countable.]

3. Consider the set $L = \{a, b, \ldots, z\}^{Z_+}$ of all infinite words in the roman alphabet. Give this set the dictionary order. Answer the following questions about L. In all cases, you **must** explain your answers. A little handwaving is OK — there isn't enough time to be completely rigorous about everything — but your reasoning should be clear.

a) Which points have immediate successors? Which have immediate predecessors?

b) Is L well-ordered? (That is, does every non-empty subset of L have a least element?)

c) Does L have the least-upper-bound property?