M427K Second Midterm Exam, March 7, 2008

1. Consider the differential equation $y'' - 2y' + 5y = 0$.

a) Find the general solution.

b) Find the solution with the initial conditions $y(0) = 1$ and $y'(0) = 5$.

c) Find a particular solution to $y'' - 2y' + 5y = 17\cos(2t)$.

2. Consider the differential equation $y''' - 3y'' + 2y' = 0$.

a) Find the general solution.

b) Find a particular solution to $y''' - 3y'' + 2y' = 6e^{-t}$.

c) Find the solution to $y''' - 3y'' + 2y' = 6e^{-t}$ with initial conditions $y(0) = 18$, $y'(0) = 25$, $y''(0) = 35$.

3a. Let A be a 4 by 6 matrix whose reduced row-echelon form is

$$A_{RREF} = \begin{pmatrix} 1 & 2 & 0 & 3 & 0 & 5 \\ 0 & 0 & 1 & 4 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Find all solutions to $A\vec{x} = 0$.

b) Consider the matrix $A = \begin{pmatrix} 1 & 1 & 1 & -2 \\ 1 & 2 & 4 & -3 \\ 2 & 4 & 9 & -7 \\ 4 & 7 & 14 & -12 \end{pmatrix}$. Are the columns of this matrix linearly independent? If so, express $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ as a linear combination of the columns of A. If not, find a nontrivial linear combination of the columns that equals zero.

4a) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 4 & 2 \\ 5 & 1 \end{pmatrix}$.

b) The matrix $\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$ has eigenvalues 1 and 8, with corresponding eigenvectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$. Find the general solution to the equations

$$\begin{aligned} x_1' &= 6x_1 + 5x_2 \\ x_2' &= 2x_1 + 3x_2. \end{aligned}$$

c) Find the solution with initial conditions $x_1(0) = 7$, $x_2(0) = 7$.