Problem 1: Find all solutions to the following system of equations:

\[
\begin{align*}
  x_1 + x_2 + x_3 - 3x_4 &= 6 \\
  2x_1 + 3x_2 - x_3 - 4x_4 &= 11 \\
  x_1 - x_2 + x_3 - x_4 &= 2 \\
  x_1 - x_2 - x_3 + x_4 &= 0
\end{align*}
\]

Problem 2

a) Is the matrix \( A = \begin{pmatrix} 1 & 6 & 3 \\ 1 & 5 & 2 \\ 0 & 2 & 1 \end{pmatrix} \) singular or non-singular? If \( A \) is non-singular, find \( A^{-1} \).

b) Find all solutions to \( AX = B \), where \( A \) is given above and \( B = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \). (Hint: Use the result of part a)

Problem 3. By doing row operations, put these matrices in reduced row-echelon form:

a) 
\[
\begin{pmatrix} 1 & 2 & 1 & 4 & 3 \\ 2 & 4 & 3 & 6 & 5 \\ -1 & -2 & 0 & 2 & 12 \end{pmatrix}
\]

b) 
\[
\begin{pmatrix} 0 & 2 & 4 \\ -2 & 0 & 2 \\ 3 & 4 & 5 \\ 1 & 2 & 3 \end{pmatrix}
\]

Problem 4. Evaluate the following determinants:

a) 
\[
\begin{vmatrix} 3 & 5 \\ 7 & 12 \end{vmatrix}
\]

b) 
\[
\begin{vmatrix} 1 & 3 & 2 \\ -1 & 1 & 5 \\ 2 & -2 & 4 \end{vmatrix}
\]

Problem 5. True of False

a) If \( A \) is a \( 3 \times 5 \) matrix that has rank 3, then the equation \( AX = B \) has at least one solution, regardless of what \( B \) is.
b) If $A$ is a $5 \times 3$ matrix that has rank 3, then the equation $AX = B$ has at least one solution, regardless of what $B$ is.

c) If $A$ is a singular $n \times n$ matrix, then $AX = 0$ has infinitely many solutions.

d) If $A$ is a nonsingular $n \times n$ matrix, then $AX = B$ has exactly one solution, namely $X = A^{-1}B$.

e) $5 2 1 4 3$ is an even permutation of $1 2 3 4 5$

f) If $A$ and $B$ are nonsingular $n \times n$ matrices, then $(AB)^{-1} = A^{-1}B^{-1}$.

g) If $AX = B$ has exactly one solution, then $AX = 0$ has exactly one solution.

h) If $AX = 0$ has infinitely many solutions, then $AX = B$ has infinitely many solutions.

i) If $AX = B$ has infinitely many solutions, then $AX = 0$ has infinitely many solutions.

j) If two rows of a square matrix are the same, the determinant of that matrix is zero.