

M346 Second Midterm Exam, October 23, 2009

1) Diagonalization:

a)(10 pts) Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 3 & 8 \\ 2 & -3 \end{pmatrix}$

b) (10 pts) Compute e^{Bt} , where $B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$.

c) (10 pts) Find the eigenvalues of $C = \begin{pmatrix} 2 & 2 & 3 & 7 \\ 2 & 5 & 2 & 8 \\ 0 & 0 & 6 & 16 \\ 0 & 0 & 4 & -6 \end{pmatrix}$. You do not need

to find the eigenvectors.

2. Consider the system of equations $\mathbf{x}(n+1) = A\mathbf{x}(n)$, where $A = \begin{pmatrix} 0 & 1 \\ 2 & -1 \end{pmatrix}$,

with initial condition $\mathbf{x}(0) = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

a) (15 pts) Find $\mathbf{x}(n)$ for all n . Be as explicit as possible.

b) (5 pts) Find $\lim_{n \rightarrow \infty} \frac{x_1(n)}{x_2(n)}$.

c) (5 pts) Find $\lim_{n \rightarrow \infty} \frac{x_1(n+1)}{x_1(n)}$.

3. a) (15 pts) Consider the system of nonlinear coupled differential equations

$$\begin{aligned} \frac{dx_1}{dt} &= x_1(3 - 2x_1 - x_2) \\ \frac{dx_2}{dt} &= x_2(3 - x_1 - 2x_2). \end{aligned}$$

This system of equations has four fixed points, namely $(0, 0)^T$, $(3/2, 0)^T$, $(0, 3/2)^T$, and $(1, 1)^T$. These equations describe competition between two species in the same environment. For each fixed point, find the matrix that describes the linearization near the fixed point, indicate how many stable, neutral, and unstable modes there are, and indicate whether the fixed point is stable, unstable, or neutral.

3 b) (10 pts) Now consider the equations

$$\begin{aligned} \frac{dx_1}{dt} &= x_1(3 - x_1 - 2x_2) \\ \frac{dx_2}{dt} &= x_2(3 - 2x_1 - x_2). \end{aligned}$$

These equations describe somewhat more intense competition, and have fixed points $(0, 0)^T$, $(3, 0)^T$, $(0, 3)^T$ and $(1, 1)^T$. For each fixed point, find the matrix that describes the linearization near the fixed point, indicate how many stable, neutral, and unstable modes there are, and indicate whether the fixed point is stable, unstable, or neutral.

4. True or False? Each question is worth 4 points. You do NOT need to justify your answers, and partial credit will NOT be given. For all questions, A is a square real matrix.

- a) The eigenvalues of A must be real.
- b) If the trace of a A is positive, then the system of equations $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$ is unstable.
- c) If the trace of a A is negative, then the system of equations $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$ is stable.
- d) The geometric multiplicity of an eigenvalue can be greater than the algebraic multiplicity.
- e) If A is a 5×5 matrix whose characteristic polynomial has 5 distinct roots, then A is diagonalizable.