1) (15 points) Consider the vectors \( \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \) and \( \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} \) in \( \mathbb{R}^3 \). Are these vectors linearly independent? Do they span \( \mathbb{R}^3 \)? Do they form a basis for \( \mathbb{R}^3 \)?

2. (15 points) Let \( V = \mathbb{R}_2[t] \) be the space of quadratic polynomials in a variable \( t \) and consider the linear transformation \( L(p) = (t + 1)p'(t) \) from \( V \) to itself, where \( p'(t) \) is the derivative of \( p(t) \). Find the matrix of this linear transformation with respect to the (standard) basis \( \{1, t, t^2\} \).

3. Let \( A = \begin{pmatrix} 1 & -1 & -1 & 1 & 8 \\ 1 & 2 & 8 & 3 & 7 \\ 1 & 2 & 8 & -2 & -28 \\ 1 & 5 & 17 & 0 & -29 \end{pmatrix} \)

   a) Find a basis for the null space of \( A \).
   
   b) Find a basis for the column space of \( A \).

4. a) In \( \mathbb{R}^2 \), let \( B = \left\{ \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 5 \\ 8 \end{pmatrix} \right\} \) be a basis, and let \( x = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \). Let \( E \) be the standard basis. Compute the change-of-basis matrices \( P_{EB} \) and \( P_{BE} \) and compute the coordinates of \( x \) in the \( B \) basis.

   b) In \( \mathbb{R}_1[t] \), let \( D = \{3 + 5t, 5 + 8t\} \) and let \( p(t) = 1 + t \). Compute \( [p]_D \).

5. a) Find the characteristic polynomial of \( \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix} \). (You do not have to compute the eigenvalues or eigenvectors).

   b) Find the eigenvalues of \( \begin{pmatrix} 1 & -4 \\ 1 & 1 \end{pmatrix} \). (You do not have to find the eigenvectors).

   c) \( \lambda = 2 \) is one of the eigenvalues of \( \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix} \). Find a basis for the corresponding eigenspace.