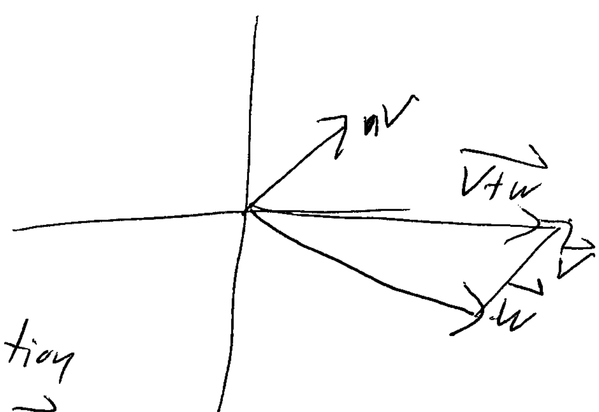
vector = list of #s = $\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$

$$\left\{\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix}\right\} = \mathbb{R}^n$$

$$c\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix} = \begin{pmatrix}cx_1\\ \vdots\\ cx_n\end{pmatrix}$$ rescale

$$\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix} + \begin{pmatrix}y_1\\ \vdots \\ y_n\end{pmatrix} = \begin{pmatrix}x_1+y_1\\ \vdots \\ x_n+y_n\end{pmatrix}$$ add.

$v=\begin{pmatrix}1\\1\end{pmatrix}$, $w=\begin{pmatrix}3\\-2\end{pmatrix}$



$3\vec{v} + 17\vec{w}$ = linear combination of $\vec{v}$ and $\vec{w}$

$$\begin{pmatrix} \text{height in inches} \\ \text{weight in lbs} \\ \text{heart rate} \end{pmatrix}$$ vector, but no notion of length

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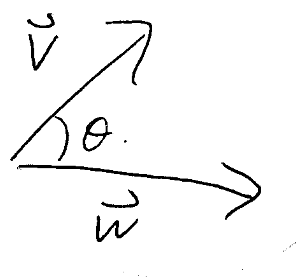
$\mathbb{R}^n = \left\{ \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \right\}$ has a notion of length

$$\vec{V} \cdot \vec{W} = \sum_i V_i W_i$$

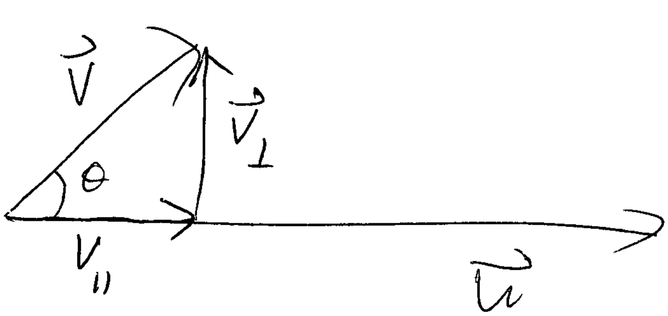
$$\vec{V} \cdot \vec{V} = \sum_i V_i^2 = V_1^2 + V_2^2 + \cdots V_n^2 = \|\vec{V}\|^2$$

(or $|\vec{V}|^2$)

$\cancel{\vec{V} \cdot \vec{W}}$



If $V \perp W$, then $\vec{V} \cdot \vec{W} = 0$.



$$\vec{V} \cdot \vec{W} = (\vec{V}_\parallel \cdot \vec{W}) + (\vec{V}_\perp \cdot \vec{W})$$
$$= |\vec{V}_\parallel| |\vec{W}|$$
$$= |\vec{V}| \cos\theta \, |\vec{w}|$$

$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \vec{w} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\vec{v} \cdot \vec{v} = 14 \qquad \vec{w} \cdot \vec{w} = 2 \qquad \vec{v} \cdot \vec{w} = -2$

$|\vec{v}| = \sqrt{14} \qquad |\vec{w}| = \sqrt{2}$

$-2 = \sqrt{14}\,\sqrt{2}\,\cos\theta$

$\cos\theta = \dfrac{-2}{\sqrt{28}} = \dfrac{-1}{\sqrt{7}}$

$\theta = \cos^{-1}\left(\dfrac{-1}{\sqrt{7}}\right)$

$m \times n$ Matrix = (1) array of #s $\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$

a_{ij} = entry in i-th row
j-th column.

(2) ordered. collection of n vectors in $\mathbb{R}^m$.

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix} = \left(\begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix} \begin{pmatrix} 2 \\ 6 \\ 10 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \\ 11 \end{pmatrix} \begin{pmatrix} 4 \\ 8 \\ 12 \end{pmatrix} \right)$$

(3) Operation on vectors.

$$(1\ 2\ 3\ 4)\begin{pmatrix}3\\1\\5\\8\end{pmatrix} = \begin{aligned} &1\cdot 3 + 2\cdot 1 + 3\cdot 5 + 4\cdot 8 \\ &= 3+2+15+32 \\ &= 52\end{aligned}$$

$$(a_1 \ \cdots \ a_n)\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix} := \begin{aligned} &a_1x_1 + \cdots \ a_nx_n \\ &= x_1a_1 + \cdots \ x_na_n\end{aligned}$$

$$A = \begin{array}{cc} & n \\ m & \begin{pmatrix}\vec{a}_1 & \cdots & \vec{a}_n\end{pmatrix}\end{array}\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix} := x_1\vec{a}_1 + \cdots \ x_n\vec{a}_n \in \mathbb{R}^m$$

= linear combination of cols of A with coefficients given by ~~x~~ entries of $\vec{x}$.

$$\begin{pmatrix}1&1&1\\1&2&3\end{pmatrix}\begin{pmatrix}3\\5\\7\end{pmatrix} = 3\begin{pmatrix}1\\1\end{pmatrix}+5\begin{pmatrix}1\\2\end{pmatrix}+7\begin{pmatrix}1\\3\end{pmatrix}$$

$$= \begin{pmatrix}3\cdot 1+5\cdot 1+7\cdot 1\\3\cdot 1+5\cdot 2+7\cdot 3\end{pmatrix} = \begin{pmatrix}15\\34\end{pmatrix}$$

<u>Thm</u>: If $A = \begin{pmatrix} r_1 \\ \vdots \\ r_m \end{pmatrix}$

Then $Ax = \begin{pmatrix} r_1 x \\ r_2 x \\ \vdots \\ r_m x \end{pmatrix}$

$$(Ax)_i = \sum_j A_{ij} x_j$$