# Myth busting @ diagonalization

1. Row-reduction has <u>nothing</u> to do with finding e-vals.

Row operations change e-vals.

E-vals are roots of $\det(A-\lambda I)$.

Tricks for finding e-vals.

<u>Algebraic</u> multiplicity $AM(\lambda)$ = # of times it shows up in list.

2. Row-reduction has <u>everything</u> to do with finding e-vectors.

Eigenspace $E_\lambda = \text{Null}(A-\lambda I)$

$GM(\lambda) = \dim E_\lambda = \dim \text{Null}(A-\lambda I) =$ # free variables

$= n -$ # pivots.

$$1 \leq GM(\lambda) \leq AM(\lambda)$$

3. Diagonalizability has nothing to do with invertibility.

Diagonalizable if and only if, for each $\lambda$, $AM = GM$.

If all AM's $= 1$, then all GM's $= 1$, diagonalizable.

If some AM's $> 1$, need to check GM's.

4) To find 3 e-vals, need 3 pieces of information!

Vector space $V$,

Basis $\mathcal{B} = \{\vec{b}_1, \dots \vec{b}_n\}$.

$\mathcal{B}$ spans $V$; $\mathcal{B}$ linearly independent.

~~If $\vec{x} \in V$, $\vec{x} =$~~

If $\vec{x} \in V$, $\vec{x} = c_1 \vec{b}_1 + \dots + c_n \vec{b}_n$

If $\vec{x} = c_1' \vec{b}_1 + \dots + c_n' \vec{b}_n$

$$0 = (c_1 - c_1')\vec{b}_1 + \dots + (c_n - c_n')\vec{b}_n.$$

Since $\mathcal{B}$ is lin ind, $c_1 - c_1' = 0 \Rightarrow c_1' = c_1$

$c_2 - c_2' = 0 \Rightarrow c_2' = c_2$

$\vdots$

There is <u>exactly</u> one way to write $\vec{x}$ as linear combination.

$$\begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix} = \text{Coordinates of } \vec{x} \text{ in the } \mathcal{B} \text{ basis}$$

$$= [\vec{x}]_{\mathcal{B}}$$

$$\begin{tikzcd}
\vec{y}(0) \arrow[r] & \vec{y}(t) \arrow[d, "S"] \\
\vec{x}(0) \arrow[u, "S^{-1}"] & \vec{x}(t)
\end{tikzcd}$$

$$\vec{x} = y_1 \vec{b}_1 + \cdots + y_n \vec{b}_n \qquad \vec{y} = [\vec{x}]_\beta$$

$$\vec{x} = S\vec{y} \qquad S = (\vec{b}_1 \cdots \vec{b}_n)$$

$$\vec{y} = S^{-1}\vec{x}$$

= Matrix that converts from $[\ ]_\beta$ to std. basis.

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Spaces that aren't $\mathbb{R}^n$.

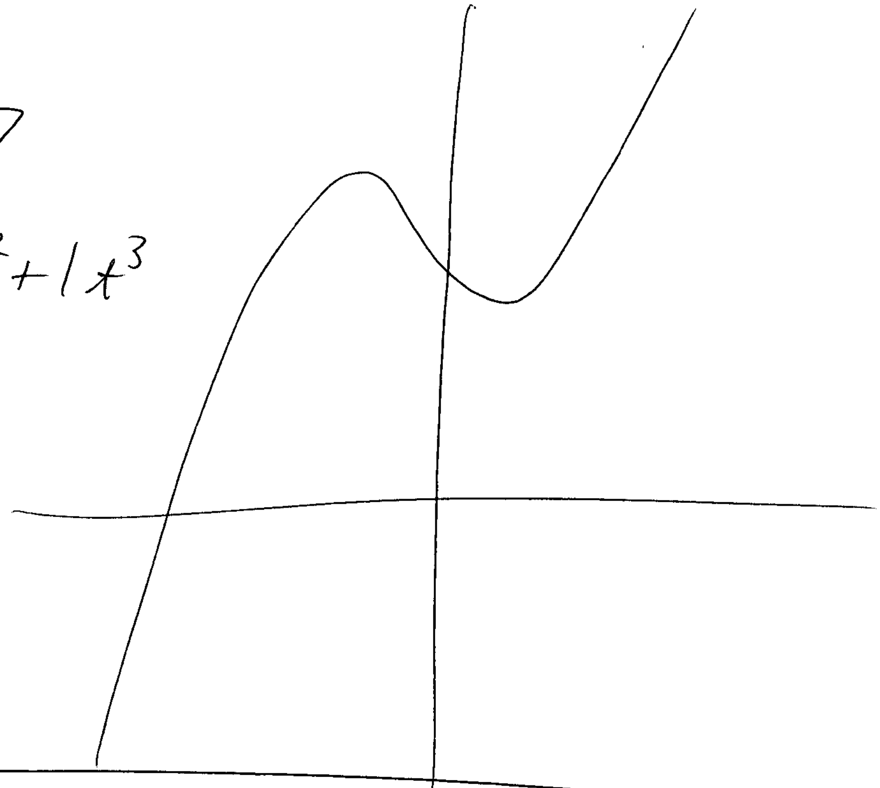
$$V = \{\text{cubic polynomials in } t\}$$
$$= \{a_0 + a_1 t + a_2 t^2 + a_3 t^3\}.$$

$$\beta = \{1, t, t^2, t^3\}$$

$$p(t) = t^3 - 3t + 7$$

$$= 7 \cdot 1 + -3t + 0t^2 + 1t^3$$

$$[p]_\beta = \begin{pmatrix} 7 \\ -3 \\ 0 \\ 1 \end{pmatrix}$$



Thm: If $V$ is a vector space with basis $\beta$ and if $\vec{x}, \vec{y} \in V$, then

$$[\vec{x} + \vec{y}]_\beta = [\vec{x}]_\beta + [\vec{y}]_\beta$$

$$[c\vec{x}]_\beta = c[\vec{x}]_\beta$$

$$V \xleftrightarrow{\ \beta\ } \mathbb{R}^n$$

$\vec{x}$ $\qquad$ $[\vec{x}]_\beta$

$\vec{x}+\vec{y}$ $\qquad$ $[\vec{x}]_\beta + [\vec{y}]_\beta$

$$\Big\updownarrow \text{ other basis } \mathcal{C}$$

$[\vec{x}]_\mathcal{C}$

$[\vec{x}]_\mathcal{C} + [\vec{y}]_\mathcal{C}$

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Ex: $\quad V =$ cubic poly.

$$\beta = \{1, t, t^2, t^3\}$$

$$\mathcal{C} = \{1, (t-1), (t-1)^2, (t-1)^3\}$$

| | | |
|---|---|---|
| 10 | 021 | 3 |
| 9 | 55161 | 5 |
| 8 | 70081, 05538, | 10 |
| 7 | 89102, 72302,<br>0213 | 14 |
| 6 | 06757, 44955,<br>28096, 16 | ~~16~~ 17 |
| 5 | 24672, 44390,<br>78458, 5 | 16 |
| 4 | 73812, 964 | 8 |
| 3 | 94047, 305 | 8 |
| 2 | 77687, 98 | 7 |
| 1 | | 0 |
| 0 | 51 | 2 |
| tot | | 90 |

Median = ~~67~~ 64

Average = 60