1. Let \( A = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 8 & 0 \\ -1 & 3 & 13 \end{pmatrix} \), and let \( b = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \).

   a) Write \( A \) as a product \( A = LU \) or \( A = PLU \), with \( L \) lower-triangular and \( U \) upper-triangular (and \( P \), if needed, a permutation matrix).

   b) Solve the system of equations \( Ax = b \).

   c) Find \( A^{-1} \). (Yes, \( A \) is invertible.)

2. Consider the vectors \( a_1 = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \), \( a_2 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \), \( a_3 = \begin{pmatrix} 7 \\ 1 \\ -7 \end{pmatrix} \), and \( v = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \).

   a) Is \( v \) in the span of the vectors \( \{a_1, a_2, a_3\} \)? If so, write \( v \) as a linear combination of \( a_1, a_2 \) and \( a_3 \). If not, explain why not.

   b) Are the vectors \( \{a_1, a_2, a_3\} \) linearly independent? If so, explain why. If not, write one of them as a linear combination of the other two.

   c) Is the matrix \( A = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \) invertible? If so, find its inverse. If not, explain why not.

3. (a) Let \( x = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \) and \( y = \begin{pmatrix} 2 \\ -2 \\ -2 \\ -2 \end{pmatrix} \). Find \( \|x\|, \|y\|, \) and \( \|x + y\| \).

   (b) Compute the angle between the vectors \( x \) and \( y \).

4. True/false. Just mark each statement with a T (or TRUE) or an F (or FALSE). You do not need to justify your answers, and partial credit will not be given.

   a) If the vectors \( a_1, a_2, a_3 \) and \( a_4 \) in \( \mathbb{R}^4 \) are linearly independent, then the matrix \( A = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \end{pmatrix} \) is invertible.

   b) A system of 3 linear equations in 5 variables always has at least one solution.

   c) If \( A \) is a singular square matrix, then there are infinitely many solutions to \( Ax = 0 \).

   d) Suppose \( A \) is a \( 3 \times 3 \) that row-reduces to an upper-triangular matrix \( U \)
by the following steps: First subtract 3 times the first row from the second, then add 2 times the first row to the third, then subtract 5 times the second row from the third. Then $A = LU$, where $L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 5 & 1 \end{pmatrix}$.

e) If $A$ and $B$ are invertible square matrices of the same size, then $AB$ is invertible and $(AB)^{-1} = A^{-1}B^{-1}$. 