M408S (Integral Calculus for Science)
Examination #1: Chapter 5: Anti-derivatives, Chapter 6: Area and Volumes, Chapter 7: Integration by Parts and Trigonometric Integrals

1. (15 points total) Area between two curves: Let one curve be the exponential function, $f(x) = 2^x$, and create a second LINEAR function of your choice such that the intersection of the two functions forms some bounded region. Hint: Graph the exponential function and choose your line so that the line passes between some nice points that you select *a priori*.

   a) (10 points) Write down your linear function and explain how you selected your function.

   b) (5 points) Set up the definite integral that represents your enclosed area. Do not attempt to compute the area.

2. (10 points) Determine the numerical value of $\int_2^7 (u+4)\sqrt{u+2}\,du$

3. (10 points) Compute $\frac{d}{dx}\left[\int_{-9}^{x^2}\left(\frac{3}{1+t^2} - \frac{1}{t} + e^{-6t}\right)dt\right]$.

4. (10 points) <u>Explain in words</u> what the following definite integral represents:

$\int_{-10}^{10} \sqrt{100 - x^2}\,dx$. Hint: Sketch a graph of the integrand.

5. (15 points) Determine the numerical value of $\int_0^4 (2 - |x-2|)dx$. Hint: Sketch a graph of the integrand.

6. (15 points total) A line passes through the points $(6,0)$ and $(0,12)$. The segment of the line from $x = 1$ to $x = 4$ is revolved around the *x*-axis forming a 3-dimensional solid of revolution. Hint: Sketch a graph of the 3-dimensional solid.

   a. (10 points) Write the definite integral that represents the volume of the solid of revolution.
   b. (5 points) Determine the numerical value of the volume of the solid of revolution.

7. (10 points) Evaluate the integral $\int w^3 \cdot \ln w \; dw$.

8. (15 points) Determine the general anti-derivative $\int \sin^3 x \cdot \cos^2 x \, dx$

Bonus (4 points) Inspired on 1/24 by suggestions from Kaitlin P,. Makenzie K., Emily S., Megan O., Sarah H., Anthony N., Charisma D, Sarah D., & Rebecca G.

Find the area of one region bounded by the curves $y = \sin(2x)$ and $y = \cos(2x)$. Hint: Sketch a graph of the two trigonometric functions.

M408S (Integral Calculus for Science)
Examination #2: Chapter 7: Trigonometric Substitution, Chapter 9: Population Growth, and Chapter 11: Sequences and Series

1. (2 points) Does $e^{\ln(2)} = \ln(e^2)$? YES NO

2. (2 points) Does $\ln(4) - 2\ln(5) = \ln\left(\frac{4}{25}\right)$? YES NO

3. (2 points) Does $(-1)^0 = 0$? YES NO

4. (2 points) If $\lim_{n\to\infty} \frac{f(x)}{g(x)}$ has indeterminate form, then $\frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$. YES NO

5. (10 points) Determine the anti-derivative of $\int \frac{2x}{\sqrt{x^2 - 9}} dx$.

6. (20 points) Find the specific solution to the differential equation $y' = \frac{x^3\sqrt{1-x^2}e^y}{y}$ subject to the initial condition $y(1) = 0$.

7. (10 points) A cult worshipping and raising honey badgers formed in 2010 (time $t = 0$). Two years later Tom Cruise joined the cult and there were 48 honey badgers. A year later, when Britney Spears joined the cult, there were 192 honey badgers in the cult. Assuming that the population of honey badgers is modeled by the law of natural growth (the rate of growth is proportional to the size of the population), how many honey badgers did the cult have when they formed in 2010?

8. (5 points) Does the following sequence converge? If so, what is its limit?

$$\left\{\left(1-\frac{2}{n}\right)^n\right\}_{n=5}^{\infty}$$

9. (10 points total)
   a. (5 points) Find an expression for the nth term of the sequence: $3, 6, 11, 18, 27, 38, \cdots$.
   b. (5 points) Find the 100^{th} term in the sequence.

10. (25 points total) Do NOT attempt to solve any of the following anti-derivatives. For each question, select an integration technique (I. – VI.) that can be used to solve the problem and indicate the key components. There may be multiple correct solutions.

    i. U-substitution with $u =$ _____________.
    ii. Partial Fractions of the form _______________.
    iii. Integration by Parts with $u =$ _______ and $dv =$ ________.
    iv. Trigonometric Substitution with $x =$ _____________.
    v. Improper Integral because of _________________.
    vi. Other (e.g. special technique, fundamental anti-derivative, graph, recognize the geometry).

A. (5 points) $\int \frac{3}{x^2-4}dx$

B. (5 points) $\int \frac{3x}{x^2-4}dx$

C. (5 points) $\int_0^5 \sqrt{25-x^2}\,dx$

D. (5 points) $\int \sin(x)\cos(x)\,dx$

E. (5 points) $\int_0^e \frac{1}{x}dx$

12. (5 points) Find the third partial sum, S_3, of the series $\sum_{n=1}^{\infty}\frac{(-1)^n \cdot 3}{\sqrt{n}}$.

13. Bonus (2 points) Determine the numerical value of $\int_1^2 \cos^2 x - 1 + \sin^2 x\,dx$

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Examination #3 (Chapter 11: Infinite Series)

1. (5 points) Determine $\lim_{n\to\infty}\left(\frac{n+2}{n}\right)^n$.

2. (15 points total) Approximate $\sqrt{26}$ by using a second degree Taylor polynomial expanded about the point $a = 25$.

   a. (12 points) That is, find $T_2(x) = \sum_{n=0}^{2} \frac{f^{(n)}(25)\cdot(x-25)^n}{n!}$ and use this polynomial to approximate $\sqrt{26}$.

   b. (3 points) Is your approximation greater than, equal to, or less than $\sqrt{26}$? Be sure to justify your answer.

3. (35 points total) For each series defined below, determine whether the series (1) converges absolutely, (2) converges conditionally or (3) diverges. Write the method that you would use to justify your answer. For example, you may write "The series converges by p-series with $p = 2 > 1$." Then stop. You will receive 1 bonus point each time you can correctly determine the exact sum of the infinite series.

   a. (5 points) $\sum_{n=0}^{\infty} \frac{1}{e^n}$

   b. (5 points) $\sum_{n=3}^{\infty} \frac{5}{n(n-1)}$.

   c. (5 points) $12 - 8 + \frac{16}{3} - \frac{32}{9} + \cdots$

   d. (5 points) $\sum_{n=0}^{\infty} \frac{1}{n!}$

   e. (5 points) $1 + \frac{1}{2\cdot\sqrt{2}} + \frac{1}{3\cdot\sqrt{3}} + \frac{1}{4\cdot\sqrt{4}} + \cdots$

   f. (5 points) $1 - .2 + \frac{.2^2}{2!} - \frac{.2^3}{3!} + \frac{.2^4}{4!} - \frac{.2^5}{5!} + \cdots$

   g. (5 points) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

4. (10 points) Use the Maclaurin series for the arctangent function to find the numerical value for $\int_0^{.1} \arctan(x)\,dx$. Your final answer can be written in the form of an infinite series. Estimate the definite integral to within $.000\overline{3} = .1^3/3$.

5. (5 points) Create your own power series whose interval of convergence is $[2,8]$.

6. (5 points) Given $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$, then find $\sum_{n=3}^{\infty} \frac{1}{(2n-1)^2}$.

7. (10 points) Determine if the series $\frac{1}{100} - \frac{1}{200} + \frac{1}{300} - \frac{1}{400} + \cdots$ converges conditionally, converges absolutely, or diverges. Justify your answer.

8. (10 points) Find the interval of convergence for the following power series. Be sure to check endpoints.

$$\sum_{n=2}^{\infty} \frac{(-1)^n (x-5)^n}{\sqrt{n} \cdot 4^n}$$

9. (5 points) Find the Maclaurin series for $f(x) = 5x^3 \cdot e^{2x}$.

Bonus (2 points): Find $f^{(10)}(0)$ for the aforementioned function.