M408S (Sp) – Integral Calculus
Midterm 1

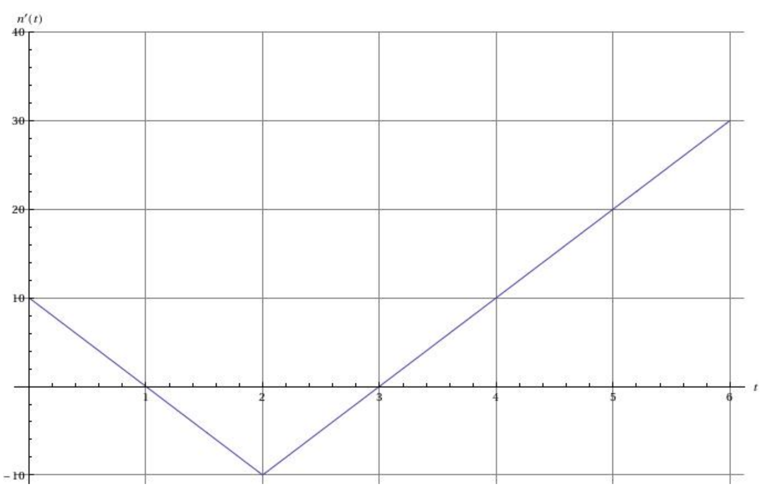
Ex # 1. A honeybee population starts with 100 bees and changes at a rate of $n'(t)$, where t is measured in weeks.

(1) What are the units of $\int_5^7 n'(t)\,dt$?

(2) What does $\int_5^7 n'(t)\,dt$ represent?

(3) What does $100 + \int_0^{10} n'(t)\,dt$ represent?

(4) Suppose that n' is given by the graph below. Find how many honeybees are in the colony at the end of the first 6 weeks.



Ex # 2. There is a popular clothing store that opens at noon. The function $f(x) = 3x + 5$ gives the rate of people entering the store per hour after noon. The function $g(x) = x + 5$ is the rate of females that enter the store per hour. How many males enter the store between 3 PM and 7 PM?

Ex # 3. Given that $f(2) = 3$, $\int_2^7 f(x)\,dx = 12$, $\int_2^3 f(x)\,dx = 5$, and $\int_2^7 f'(x)\,dx = 3$, compute the following quantities. (Write "NI" if there's not enough information.)

(1) $f(7) =$

(2) $\int_2^7 x f'(x)\, dx =$

(3) $\int_1^6 f'(x+1)\, dx =$

(4) $\int_2^3 x f(x^2-2)\, dx =$

Ex # 4. Evaluate the following integrals.

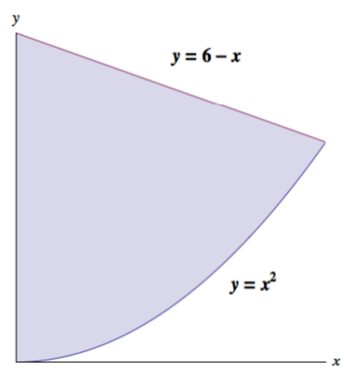
(1) $\displaystyle\int (5x+5)e^{x^2+2x+3}\, dx =$

(2) $\displaystyle\int \cos x \ln(\sin x)\, dx =$

(3) $\displaystyle\int \frac{\sin x \cos x}{\sec^2 x}\, dx =$

(4) $\displaystyle\int_0^{\pi/2} 2\left(\cos^3\theta - \cos\theta\right)\, d\theta =$

Ex # 5. The shaded region below is bounded by the curves $y = x^2$, $y = 6 - x$ and $x = 0$.



(1) Set up (but do not evaluate) an integral that would give the volume of the solid obtained by rotating this region about the y-axis.

(2) Set up (but do not evaluate) an integral that would give the volume of the solid obtained by rotating this region about the x-axis.

(3) Set up (but do not evaluate) a *different* integral that would give the volume of the solid obtained by rotating this region about the y-axis.

Ex # 6. Consider

$$\int \frac{x^2 - 4}{(x^2+3)(7-x)(x+1)^3}\,dx.$$

Write down the partial-fractions decomposition of the integrand. (You need not compute the coefficients that occur.)

Ex # 7. The following integral

$$\int_{-\infty}^{0} \frac{1}{\sqrt{8-x}}\,dx$$

☐ converges, ☐ diverges.

Justify your answer.

Ex # 8.

(1) The following series

☐ converges absolutely, ☐ converges conditionally, ☐ diverges.

Justify your answer.

$$\sum_{j=1}^{\infty} (-1)^j \frac{2\sqrt{j}}{3+j}$$

(2) The following series

☐ converges absolutely, ☐ converges conditionally, ☐ diverges

Justify your answer.

$$\sum_{n=2}^{\infty} \frac{(n-1)!}{5^n}$$

(3) The following series

☐ converges absolutely, ☐ converges conditionally, ☐ diverges.

Justify your answer.

$$\sum_{n=3}^{\infty} \frac{6+2^n}{7^n}$$

Ex # 9. Let $h(t)$ be the population of humans in Austin at time t in days. Write down a differential equation that models the growth of h as a function of time t in days if:

(1) the only factor affecting the human population is constant growth.

(2) in addition to (1), the human population dies at a rate that it's proportional to the population size.

(3) in addition to (2), zombies attack humans. Make sure that your model also includes the zombie population z (and remember that zombies need humans to survive!).

Ex # 10. The n^{th} partial sum S_n of an infinite series is

$$S_n = \frac{n^2 + (-1)^n}{\cos\left(\frac{1}{n}\right) + 3n^2}.$$

(1) The series ☐ converges, ☐ diverges.

(2) If it converges, then to what does it converge? If it diverges, then why? Justify your answer.

Ex # 11. Consider the differential equation

$$\frac{dx}{dt} = \frac{x \ln x}{2t}$$

for $t > 0$.

(1) Find **explicitely** all solutions of the differential equation.

(2) Find the particular solution satisfying $x(a) = b$.

Ex # 12 (Bonus)**.** Does the following sequence converge?

$$a_n = \int_{\frac{1}{n}}^{1} \frac{1}{x^3}\,dx$$