M408S First Midterm Exam, February 21, 2013

1 (12 pts) A person drives her car from noon to 4PM at a speed of $50 + 4t + 10(\sin(\pi t))$ miles per hour, where t is the number of hours since noon. (I.e., $t = 2.5$ means 2:30 PM). At 1PM, the odometer reads 29850 miles. What does the odometer read at 3PM? (Yes, you can assume that the odometer is accurate!)

2) (10 pts) a) Compute $\sin(\tan^{-1}(3))$

(b) Compute $\cot(\sin^{-1}(3/5))$

3) (10 pts) Compute $\int 4x \sin(2x)dx$.

4) (10 pts) Compute $\displaystyle\int \frac{dx}{(1+x^2)^{3/2}}$

5) (10 pts) Compute $\displaystyle\int_0^{\pi/2} \sin^3(x)\cos^3(x)dx$.

6) (10 pts) Compute $\displaystyle\int_2^3 \frac{2dx}{x^3 - x}$

7) (10 pts) Compute $\displaystyle\int \frac{2e^{2x} + \sec^2(x)}{(e^{2x} + \tan(x))^2}dx$

8) (10 pts) For what real values of p does the integral $\int_1^\infty \frac{dx}{x^p}$ converge? For what values does $\int_0^1 \frac{dx}{x^p}$ converge? For what values does $\int_0^\infty \frac{dx}{x^p}$ converge? Be sure to justify your answers.

9 (18 pts) Let R_1 be the region between the curve $y = \ln(x + 1)$, the x axis, and the line $x = e^2 - 1$. Let R_2 be the region between the curve $y = \ln(x+1)$, the y axis, and the line $y = 2$. In this problem you get part credit for setting up the integrals correctly and full credit for setting them up and evaluating them correctly.

a) Compute the area of R_1 by slicing vertically.

b) Compute the area of R_2 by slicing horizontally.

c) Find the volume of the region obtained by rotating R_2 around the y axis.