

M408S Second Midterm Exam, March 21, 2013

1 (30 points) This problem is about methods of integration. For each integral, you need to explain which technique you would use to do the integral, and provide a few details to back that up. **You do not have to finish the computation!** If an integral can be solved in multiple ways, then either correct answer will be accepted.

If you say “u-substitution”, indicate what u and du are, and rewrite the integral in terms of u . (Then STOP.)

If you say “integrate by parts”, indicate what u and v (not just u and dv) are. (Then STOP.)

If you say “trig integral”, indicate what trig identities you are using, and how. E.g. you might say “use $\sec^2(x) = 1 + \tan^2(x)$ to convert all but two of the secants to cosines so we can do a u-sub with $u = \tan(x)$ ”. (Then STOP.)

If you say “trig substitution”, indicate which substitution you are using and draw the relevant triangle. (Then STOP. Do not attempt the resulting trigonometric integral.)

If you say “partial fractions”, write the integrand as a sum of terms with unknown coefficients. E.g., you might say “convert the integrand to $\frac{A}{x-3} + \frac{B}{x+7}$.” (Then STOP. You do NOT have to compute the coefficients A , B , etc. or do the resulting integrals.)

If an integral can't be done in any of these ways, say “none of these”.

a) $\int \frac{2x \sec^2(\ln(x^2 + 5))}{x^2 + 5} dx$

b) $\int \frac{3x^3 + 173x^2 + 4x - 314159}{x^4 + 4x^2} dx$

c) $\int (x^2 - 9)^{5/2} dx$

d) $\int x^2 \ln(x) dx$

e) $\int \sin^4(x) \cos^7(x) dx$

2) (20 points) Consider the differential equation $\frac{dy}{dx} = y \cos(x)$.

(a) Find the general solution to this differential equation.

b) If $y(0) = 5$, what is $y(\pi/2)$?

3) (15 points) [Please give exact answers. You can leave them in terms of

exponentials and logs. E.g., an answer might be $5 \ln(1/3) - e^{0.1}$. Except that isn't actually a correct answer.] A colony of bacteria is growing exponentially. At noon there are 1,000 bacteria. An hour later, there are 3,000 bacteria.

a) How long does it take for the population to double?

b) Find the population t hours after noon, as a function of t .

c) At what time will the population hit 5,000?

4)(20 points) a) Does $\sum_{n=0}^{\infty} \frac{3^n - 2^n}{4^n}$ converge? If so, to what? If not, why not?

b) Does the sequence $\{n \sin(2\pi/n)\}$ converge? If so, to what? If not, why not?

c) Does the series $\sum_{n=1}^{\infty} n \sin(2\pi/n)$ converge? If so, why? If not, why not? (You are **not** expected to evaluate the sum.)

5) (15 points) Indicate whether each of these integrals converges, and why it does or doesn't. You don't need to say what it converges to.

a) $\int_0^{\infty} 5xe^{-x^2} dx$

b) $\int_0^{\pi} \sec^2(x) dx$

c) $\int_1^{\infty} \frac{\ln(x)}{x} dx$