M408M First Midterm Exam, October 8, 2015

1) (30 points, 2 pages) A case study in ellipses. Let C be the ellipse whose equation in Cartesian coordinates is

$$\frac{x^2}{4} + y^2 = 1.$$

a) Sketch the ellipse.

b) Find a parametrization of this ellipse. (There is more than one right answer, by the way.)

c) Using your result from (b), write down an integral that equals the perimeter of the ellipse. Simplify your integrand as much as possible, but **do not attempt to compute the integral!** (This is called an "elliptic integral" and cannot be computed in closed form.)

d) Find the locations of the foci of the ellipse, and mark the foci on your sketch.

e) Compute the eccentricity of the ellipse.

f) Find the equation of a directrix of the ellipse, and draw the directrix in your sketch. (There are two directrices, one for each focus. You just need to find one of them.)

2. (25 points, 2 pages) Polar coordinates. Let S be the polar curve $r = 2\cos(\theta)$, and let C be the circle $r = 1$. (Note: if you get stuck on (a) or (b), don't give up. You may still be able to work (c), (d), and (e).)

a) Sketch the curve S.

b) Find the equations of S in Cartesian coordinates.

c) Find the slope of the line tangent to S at $\theta = \pi/4$.

d) Find the points where C and S intersect. You can express your answer either in polar or cartesian coordinates

e) Write down an explicit integral that gives the area of the region that is inside S but outside of C. You should simplify the integrand and specify the limits of integration, but you **do not** need to evaluate the integral.

3. (25 pts, 2 pages) Lines and planes.

Let $P(1,-1,1)$, $Q(3,1,4)$, $R(1,0,3)$ and $S(5,1,2)$ be points in $\mathbb{R}^3$. Let L be the line through P and Q, and let T be the plane through P, Q and R.

a) Find the equation of L. Express your answer both as a parametrization, and separately as a set of equations that x, y, z satisfy.

b) Find a vector normal to the plane T.

c) Find the equation of the plane T.

d) Find the distance from S to T.

4. (20 pts) Surfaces. Identify whether each of these surfaces is a hyperboloid of one sheet, a hyperboloid of two sheets, an elliptic paraboloid, a hyperbolic paraboloid, or an ellipsoid. [Hint: there is at most one of each. Also, you may want to complete some squares.] No justification needed. No penalty for guessing. 5 points for each correct answer.

a) $-x^2 + 2x + 2y^2 - 4z = 0$

b) $-x^2 + 2x + 2y^2 + z^2 - 4z = 0$

c) $x^2 - 2x + 2y^2 + z^2 - 4z = 0$

d) $-x^2 + 6x + 2y^2 + z^2 - 4z = 0$