M408D Second Midterm Exam, April 1, 2016

1) (20 pts) Consider the polar curve $r = e^\theta$, running from $\theta = 0$ to $\theta = \pi$.

a) Find the slope of a line tangent to this curve at $\theta = \pi/2$.

b) Find the arc-length of this curve.

c) Find the area between this curve and the x-axis.

2. (20 points) Let C be a conic section with ellipticity $e = 1/2$, focus at the origin, and whose directrix is the line $x = -2$.

a) Find the equation of C *in polar coordinates.*

b) Find an equation of C *in cartesian coordinates.*

c) If C is an ellipse or a hyperbola, find the location of the center and the parameters a and b. If C is a parabola, find the location of the vertex.

3. (20 pts) Sequences. Evaluate the following limits. Show your work!

a) $\lim_{n\to\infty} \frac{n^3}{e^n}$

b) $\lim_{n\to\infty} \frac{2n^2 + n + 5e^{-n}}{n^2 + e^{-n}}$

c) $\lim_{n\to\infty} \frac{2n^2 + n + 5e^{n}}{n^2 + e^{n}}$

d) $\lim_{n\to\infty} (3n^2 + 7)\sin(1/n)$

4. (2 pages, 40 points) For each of the following series, indicate whether the series converges absolutely, converges conditionally, or diverges, **and why.** (E.g. "converges absolutely by the ratio test with $R = 1/2$" or "diverges by comparison to $\sum 2^n$" or "converges absolutely by limit comparison to $\sum 1/n^5$")

a) $\sum_{n=1}^{\infty} \frac{n^2 + n - 7}{n^4 - n^3 + 1}$

b) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n}$

c) $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{10^n}$

d) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$

e) $\sum_{n=1}^{\infty} \frac{n^{100} 100^n}{n!}$.