M346 Third Midterm Exam, November 20, 2009

1) Gram Schmidt:

a)(10 points) On $\mathbb{R}^3$ with the usual inner product, Use Gram-Schmidt to convert $\mathbf{x}_1 = (1,2,0)^T$, $\mathbf{x}_2 = (3,1,1)^T$, $\mathbf{x}_3 = (4,3,-5)^T$ to an orthogonal basis.

b)(15 points) On $\mathbb{R}_2[t]$ with the inner product $\langle f|g\rangle = \int_0^2 f(t)g(t)dt$, transform $\{1, t, t^2\}$ to an orthogonal basis.

2. a)(15 points) Find the equation of the best line through the points $(1,-4)$, $(2,1)$, and $(3,2)$.

b)(10 points) Let V be the subspace of $\mathbb{R}^3$ that is the span of the vectors $(1,2,3)^T$ and $(1,1,1)^T$. Find the point in V that is closest to $(-4,1,2)^T$.

3. On $\mathbb{C}^3$ with the usual inner product, let

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + ix_2 - ix_3 \\ 2x_2 + (1-i)x_3 \\ ix_1 + 3x_2 + x_3 \end{pmatrix}$$

a)(5 points) Find the matrix of L

b)(10 points) Let $\mathbf{x} = \begin{pmatrix} 1 \\ 10 \\ 100 \end{pmatrix}$. Compute $L^\dagger(\mathbf{x})$.

c)(10 points) Let V be the space of real-valued functions on the real line, with the inner product $\langle f|g\rangle = \int_{-\infty}^{\infty} f(t)g(t)dt$. Let $A: V \to V$ be the linear transformation $A = t + d/dt$ (That is, $(A(f))(t) = tf(t) + f'(t)$). Let $g(t) = e^{-t^2/2}$. Compute Ag and $A^\dagger g$.

4. Grab bag. These are short-answer or true/false questions. Each question is worth 5 points. You do NOT need to justify your answers, and partial credit will NOT be given.

a) True or false? The matrix $\begin{pmatrix} 5 & 4i \\ -4i & -1 \end{pmatrix}$ has orthogonal eigenvectors.

b) True or false? The matrix $\frac{1}{\sqrt{7}}\begin{pmatrix} 2-i & -1+i \\ 1+i & 2+i \end{pmatrix}$ is unitary.

c) Let $\mathbf{x}(t)$ be the solution to $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$, where $A = \begin{pmatrix} 0 & 1 & 2 & 3 \\ -1 & 0 & -1 & 4 \\ -2 & 1 & 0 & 5 \\ -3 & -4 & -5 & 0 \end{pmatrix}$ and $\mathbf{x}(0) = (5,-3,1,1)^T$ Find the limit, as $t \to \infty$, of $|\mathbf{x}(t)|$. (This has a quick and easy solution, and you do NOT have to diagonalize A!)

d) True or false? If a matrix M satisfies $M = M^T$, then the eigenvalues of M are real.

e) True or false? If a matrix is unitary, then it is not Hermitian.