1) (25 pts) Consider the matrix $A = \begin{pmatrix} 4 & 2 \\ 8 & -2 \end{pmatrix}$.
   a) Find the eigenvalues of $A$. For each eigenvalue, find a corresponding eigenvector.
   b) Compute $A^n$ for all $n$. Make your answer as explicit as possible.
   c) Compute $e^{At}$ as a function of $t$. Make your answer as explicit as possible.

2. (30 points, 2 pages) Let $A = \begin{pmatrix} -4 & 5 \\ 5 & -4 \end{pmatrix}$. This matrix has eigenvalues 1 and $-9$ and corresponding eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.
   a) Find the solution to $x(n+1) = Ax(n)$ with initial condition $x(0) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.
   b) Find the solution to $\frac{dx}{dt} = Ax$ with initial condition $x(0) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.
   c) Find the solution to $\frac{d^2x}{dt^2} = Ax$ with initial conditions $x(0) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\dot{x}(0) = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$.

3. (15 pts) The differential equations
   \[
   \frac{dx_1}{dt} = e^{-4x_1} - x_2 \\
   \frac{dx_2}{dt} = 5x_1x_2
   \]
   have a fixed point at $x_1 = 0, x_2 = 1$. Find the linear approximation to these equations near $x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and determine whether this fixed point is stable. Explain your reasoning!
4. (15 points) On $\mathbb{R}_2[t]$ with the inner product $\langle f|g \rangle = \int_0^1 f(t)g(t)dt$, use Gram-Schmidt to convert $\{1, 2t, 6t^2\}$ to an orthogonal basis.

5. (15 points) In $\mathbb{R}^5$ with the standard inner product, let $V$ be the subspace spanned by

\[
\begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 4 \\ 3 \\ 5 \\ -2 \\ 1 \end{pmatrix}.
\]

Write $b = \begin{pmatrix} 7 \\ 4 \\ 5 \\ 1 \\ 2 \end{pmatrix}$ as the sum of two vectors, one in $V$ and the other orthogonal to $V$. 