M346 First Midterm Exam, February 17, 2011

1) (15 points) Consider the vectors $\begin{pmatrix}1\\1\\3\end{pmatrix}$, $\begin{pmatrix}1\\2\\5\end{pmatrix}$ and $\begin{pmatrix}3\\1\\5\end{pmatrix}$ in $\mathbb{R}^3$. Are these vectors linearly independent? Do they span $\mathbb{R}^3$? Do they form a basis for $\mathbb{R}^3$?

2. (15 points) Let $V = \mathbb{R}_2[t]$ be the space of quadratic polynomials in a variable t and consider the linear transformation $L(\mathbf{p}) = (t+1)\mathbf{p}'(t)$ from V to itself, where $\mathbf{p}'(t)$ is the derivative of $\mathbf{p}(t)$. Find the matrix of this linear transformation with respect to the (standard) basis $\{1, t, t^2\}$.

3. Let $A = \begin{pmatrix}1 & -1 & -1 & 1 & 8\\ 1 & 2 & 8 & 3 & 7\\ 1 & 2 & 8 & -2 & -28\\ 1 & 5 & 17 & 0 & -29\end{pmatrix}$. A is row-equivalent to $\begin{pmatrix}1 & 0 & 2 & 0 & -4\\ 0 & 1 & 3 & 0 & -5\\ 0 & 0 & 0 & 1 & 7\\ 0 & 0 & 0 & 0 & 0\end{pmatrix}$.

a) Find a basis for the null space of A.

b) Find a basis for the column space of A.

4. a) In $\mathbb{R}^2$, let $\mathcal{B} = \left\{\begin{pmatrix}3\\5\end{pmatrix}, \begin{pmatrix}5\\8\end{pmatrix}\right\}$ be a basis, and let $\mathbf{x} = \begin{pmatrix}1\\-2\end{pmatrix}$. Let $\mathcal{E}$ be the standard basis. Compute the change-of-basis matrices $P_{\mathcal{E}\mathcal{B}}$ and $P_{\mathcal{B}\mathcal{E}}$ and compute the coordinates of $\mathbf{x}$ in the $\mathcal{B}$ basis.

b) In $\mathbb{R}_1[t]$, let $\mathcal{D} = \{3+5t, 5+8t\}$ and let $\mathbf{p}(t) = 1+t$. Compute $[\mathbf{p}]_{\mathcal{D}}$.

5. a) Find the characteristic polynomial of $\begin{pmatrix}3 & 2\\ 5 & 1\end{pmatrix}$. (You do not have to compute the eigenvalues or eigenvectors).

b) Find the eigenvalues of $\begin{pmatrix}1 & -4\\ 1 & 1\end{pmatrix}$. (You do not have to find the eigenvectors).

c) $\lambda = 2$ is one of the eigenvalues of $\begin{pmatrix}3 & 1 & 1\\ 1 & 3 & 1\\ 1 & 1 & 3\end{pmatrix}$. Find a basis for the corresponding eigenspace.