# M427J: Differential Equations with Linear Algebra
# Homework # 01
# Handout: 02/17/2017, Tuesday
# Due: 02/25/2017, Wednesday

- **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Wednesday** in the Problem Session. Note that *no late homework will be accepted*.

- **Assignments for Section 1.1: Differential Equations and Their Solutions**

In each of Problems 1 through 6 determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. $t^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin t$
2. $(1+y^2)\frac{d^2y}{dt^2} + t\frac{dy}{dt} + y = e^t$
3. $\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 1$
4. $\frac{dy}{dt} + ty^2 = 0$
5. $\frac{d^2y}{dt^2} + \sin(t+y) = \sin t$
6. $\frac{d^3y}{dt^3} + t\frac{dy}{dt} + (\cos^2 t)y = t^3$

In each of Problems 7 through 10 verify that each given function is a solution of the differential equation.

7. $y'''' + 4y''' + 3y = t;$ $\quad y_1(t) = t/3, \quad y_2(t) = e^{-t} + t/3$
8. $2t^2y'' + 3ty' - y = 0,\ t > 0;$ $\quad y_1(t) = t^{1/2}, \quad y_2(t) = t^{-1}$
9. $t^2y'' + 5ty' + 4y = 0,\ t > 0;$ $\quad y_1(t) = t^{-2}, \quad y_2(t) = t^{-2}\ln t$
10. $y'' + y = \sec t,\ 0 < t < \pi/2;$ $\quad y(t) = (\cos t)\ln\cos t + t\sin t$

In each of Problems 11 through 14 determine the order of the given partial differential equation; also state whether the equation is linear or nonlinear. Partial derivatives are denoted by subscripts.

11. $u_{xx} + u_{yy} + u_{zz} = 0$
12. $u_{xx} + u_{yy} + uu_x + uu_y + u = 0$
13. $u_{xxxx} + 2u_{xxyy} + u_{yyyy} = 0$
14. $u_t + uu_x = 1 + u_{xx}$

In each of Problems 15 through 18 verify that each given function is a solution of the partial differential equation. $\lambda$ and $\alpha$ are constants.

15. $u_{xx} + u_{yy} = 0;$ $\quad u_1(x,y) = \cos x \cosh y, \quad u_2(x,y) = \ln(x^2+y^2)$
16. $\alpha^2 u_{xx} = u_t;$ $\quad u_1(x,t) = e^{-\alpha^2 t}\sin x, \quad u_2(x,t) = e^{-\alpha^2\lambda^2 t}\sin\lambda x$
17. $a^2 u_{xx} = u_{tt};$ $\quad u_1(x,t) = \sin\lambda x \sin\lambda at, \quad u_2(x,t) = \sin(x - at),$
18. $\alpha^2 u_{xx} = u_t;$ $\quad u(x,t) = (\pi/t)^{1/2} e^{-x^2/4\alpha^2 t},\ t > 0$

- **Assignments for Section 1.2: The First-Order Linear ODEs**

In each of Problems 1 through 7 find the solution of the initial value problem.

1. $y' - y = 2te^{2t}, \qquad y(0) = 1$
2. $y' + 2y = te^{-2t}, \qquad y(1) = 0$
3. $ty' + 2y = t^2 - t + 1, \qquad y(1) = 1/2, \quad t > 0$
4. $y' + (2/t)y = (\cos t)/t^2, \qquad y(\pi) = 0, \quad t > 0$
5. $y' - 2y = e^{2t}, \qquad y(0) = 2$
6. $ty' + 2y = \sin t, \qquad y(\pi/2) = 1, \quad t > 0$
7. $t^3y' + 4t^2y = e^{-t}, \qquad y(-1) = 0, \quad t < 0$

8. Find the value of $y_0$ for which the solution of the initial value problem

$$y' - y = 1 + 3\sin t, \qquad y(0) = y_0$$

remains finite as $t \to \infty$.

9. **Variation of Parameters.** Consider the following method of solving the general linear equation of first order:

$$y' + p(t)y = g(t). \tag{I}$$

(a) If $g(t) = 0$ for all $t$, show that the solution is

$$y = A\exp\Big[-\int p(t)dt\Big], \tag{II}$$

where $A$ is a constant.
(b) If $g(t)$ is not everywhere zero, assume that the solution of equation (I) if of the form

$$y = A(t)\exp\Big[-\int p(t)dt\Big], \tag{III}$$

where $A$ is now a function of $t$. By substituting for $y$ in the given differential equation, show that $A(t)$ must satisfy the condition

$$A'(t) = g(t)\exp\Big[\int p(t)dt\Big]. \tag{IV}$$

(c) Find $A(t)$ from equation (IV). Then substitute for $A(t)$ in equation (III) and determine $y(t)$. This technique is known as the method of **variation of parameters**.