• Submission: Please make your homework neat and **STAPLED**. You have to submit **Wednesday** in the Problem Session. Note that **no late homework will be accepted**.

• Assignments for Section 1.1: Differential Equations and Their Solutions

In each of Problems 1 through 6 determine the order of the given differential equation; also state whether the equation is linear or nonlinear.

1. \( t^2 \frac{d^2y}{dt^2} + t \frac{dy}{dt} + 2y = \sin t \)
2. \( (1 + y^2) \frac{d^2y}{dt^2} + t \frac{dy}{dt} + y = e^t \)
3. \( \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 1 \)
4. \( \frac{dy}{dt} + ty^2 = 0 \)
5. \( \frac{d^2y}{dt^2} + \sin(t + y) = \sin t \)
6. \( \frac{d^3y}{dt^3} + t \frac{dy}{dt} + (\cos^2 t)y = t^3 \)

In each of Problems 7 through 10 verify that each given function is a solution of the differential equation.

7. \( y''' + 4y'' + 3y = t; \quad y_1(t) = t/3, \quad y_2(t) = e^{-t} + t/3 \)
8. \( 2t^2y'' + 3ty' - y = 0, \quad t > 0; \quad y_1(t) = t^{1/2}, \quad y_2(t) = t^{-1} \)
9. \( t^2y'' + 5ty' + 4y = 0, \quad t > 0; \quad y_1(t) = t^{-2}, \quad y_2(t) = t^{-2} \ln t \)
10. \( y'' + y = \sec t, \quad 0 < t < \pi/2; \quad y(t) = (\cos t) \ln \cos t + t \sin t \)

In each of Problems 11 through 14 determine the order of the given partial differential equation; also state whether the equation is linear or nonlinear. Partial derivatives are denoted by subscripts.

11. \( u_{xx} + u_{yy} + u_{zz} = 0 \)
12. \( u_{xx} + u_{yy} + uu_x + uu_y + u = 0 \)
13. \( u_{xxxx} + 2u_{xxyy} + u_{yyyy} = 0 \)
14. \( u_t + uu_x = 1 + u_{xx} \)

In each of Problems 15 through 18 verify that each given function is a solution of the partial differential equation. \( \lambda \) and \( \alpha \) are constants.

15. \( u_{xx} + u_{yy} = 0; \quad u_1(x, y) = \cos x \cosh y, \quad u_2(x, y) = \ln(x^2 + y^2) \)
16. \( \alpha^2 u_{xx} = u_t; \quad u_1(x, t) = e^{-\alpha^2 \lambda^2 t} \sin x, \quad u_2(x, t) = e^{-\alpha^2 \lambda^2 t} \sin \lambda x \)
17. \( \alpha^2 u_{xx} = u_{tt}; \quad u_1(x, t) = \sin \lambda x \sin \lambda \alpha t, \quad u_2(x, t) = \sin(x - \alpha t), \quad \alpha^2 u_{xx} = u_t; \quad u(x, t) = (\pi/t)^{1/2} e^{-x^2/4\alpha^2 t}, \quad t > 0 \)
Assignments for Section 1.2: The First-Order Linear ODEs

In each of Problems 1 through 7 find the solution of the initial value problem.

1. \( y' - y = 2te^{2t}, \quad y(0) = 1 \)
2. \( y' + 2y = te^{-2t}, \quad y(1) = 0 \)
3. \( ty' + 2y = t^2 - t + 1, \quad y(1) = 1/2, \quad t > 0 \)
4. \( y' + (2/t)y = (\cos t)/t^2, \quad y(\pi) = 0, \quad t > 0 \)
5. \( y' - 2y = e^{2t}, \quad y(0) = 2 \)
6. \( ty' + 2y = \sin t, \quad y(\pi/2) = 1, \quad t > 0 \)
7. \( t^3y' + 4t^2y = e^{-t}, \quad y(-1) = 0, \quad t < 0 \)

8. Find the value of \( y_0 \) for which the solution of the initial value problem
   \[ y' - y = 1 + 3\sin t, \quad y(0) = y_0 \]
   remains finite as \( t \to \infty \).

9. **Variation of Parameters.** Consider the following method of solving the general linear equation of first order:
   \[ y' + p(t)y = g(t). \] \hspace{1cm} (I)

   (a) If \( g(t) = 0 \) for all \( t \), show that the solution is
   \[ y = A \exp \left[ -\int p(t)dt \right], \] \hspace{1cm} (II)
   where \( A \) is a constant.

   (b) If \( g(t) \) is not everywhere zero, assume that the solution of equation (I) if of the form
   \[ y = A(t) \exp \left[ -\int p(t)dt \right], \] \hspace{1cm} (III)
   where \( A \) is now a function of \( t \). By substituting for \( y \) in the given differential equation, show that \( A(t) \) must satisfy the condition
   \[ A'(t) = g(t) \exp \left[ \int p(t)dt \right]. \] \hspace{1cm} (IV)

   (c) Find \( A(t) \) from equation (IV). Then substitute for \( A(t) \) in equation (II) and determine \( y(t) \). This technique is known as the method of **variation of parameters**.