M427J: Differential Equations with Linear Algebra
Homework # 02
Handout: 01/24/2016, Tuesday
Due: 02/01/2016, Wednesday

• **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Wednesday** in the Problem Session. Note that *no late homework will be accepted.*

• **Assignments for Section 1.4 (Separable Equations):**

In each of the following problems solve the given differential equation.

1. $y' + y^2 \sin x = 0$
2. $y' = (3x^2 - 1)/(3 + 2y)$
3. $xy' = (1 - y^2)^{1/2}$
4. $\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}$

5. Solve the initial value problem

$$y' = 3x^2/(3y^2 - 4), \qquad y(1) = 0.$$

6. Solve the initial value problem

$$y' = 2y^2 + xy^2, \qquad y(0) = 1$$

and determine where the solution attains its minimum value.

Homogeneous Equations. If the right hand side of the equation $dy/dx = f(x, y)$ can be expressed as a function of the ratio y/x only, then the equation is said to be homogeneous. Such equations can always be transformed into separable equations by a change of the dependent variable.

7. Consider the equation

$$\frac{dy}{dx} = \frac{y - 4x}{x - y} \tag{1}$$

(a) Show that equation (1) can be rewritten as

$$\frac{dy}{dx} = \frac{(y/x) - 4}{1 - (y/x)}; \tag{2}$$

thus equation (1) is homogeneous.
(b) Introduce a new dependent variable v so that $v = y/x$, or $y = xv(x)$. Express dy/dx in terms of x, v and dv/dx.
(c) Replace y and dy/dx in equation (2) by the expressions from part (b) that involve v and dv/dx. Show that the resulting differential equation is

$$v + x\frac{dv}{dx} = \frac{v-4}{1-v}, \tag{3}$$

or

$$x\frac{dv}{dx} = \frac{v^2-4}{1-v}. \tag{4}$$

Observe that equation (4) is separable.
(d) Solve equation (4), obtaining v implicitly in terms of x.
(e) Find the solution of equation (1) by replacing v by y/x in the solution in part (d).

The method outlined in Problem 7 can be used for any homogeneous equation. That is, the substitution $y = xv(x)$ transforms a homogeneous equation into a separable equation. The latter equation can be solved by direct integration, and then replacing v by y/x gives the solution to the original equation. In each of Problems 8 through 9:
(a) Show that the given equation is homogeneous.
(b) Solve the differential equation.

8. $\dfrac{dy}{dx} = \dfrac{x^2+xy+y^2}{x^2}$ 9. $\dfrac{dy}{dx} = \dfrac{x^2+3y^2}{2xy}$

- **Assignments for Section 1.9 (Exact Equations I):**

Determine whether each of the following equations is exact. If it is exact, find the solution.

1. $(2x+4y)+(2x-2y)y' = 0$
2. $(2xy^2+2y)+(2x^2y+2x)y' = 0$
3. $\dfrac{dy}{dx} = -\dfrac{ax-by}{bx-cy}$
4. $(e^x \sin y + 3y)dx - (3x - e^x \sin y)dy = 0$
5. $(y/x+6x)dx + (\ln x - 2)dy = 0, \qquad x > 0$
6. $\dfrac{xdx}{(x^2+y^2)^{3/2}} + \dfrac{ydy}{(x^2+y^2)^{3/2}} = 0$

In each of the following problems find the value of b for which the given equation is exact, and then solve it using that value of b.

7. $(xy^2+bx^2y)dx + (x+y)x^2dy = 0$
8. $(ye^{2xy}+x)dx + bxe^{2xy}dy = 0$