M427J: Differential Equations with Linear Algebra
Homework # 03
Handout: 01/31/2017, Tuesday
Due: 02/08/2017, Wednesday

• **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Wednesday** in the Problem Session. Note that *no late homework will be accepted.*

• **Assignments for Section 1.9 (Exact Equations II):**

Show that the following equations are not exact but become exact when multiplied by the given integrating factor. Then solve the equations.

1. $x^2y^3 + x(1+y^2)y', \qquad \mu(x,y) = 1/xy^3$
2. $ydx + (2x - ye^y)dy = 0, \qquad \mu(x,y) = y$
3. $(x+2)\sin y dx + x\cos y dy = 0, \qquad \mu(x,y) = xe^x$

In each of the following problems, find an integrating factor and solve the given equation.

4. $(3x^2y + 2xy + y^3)dx + (x^2 + y^2)dy = 0$
5. $dx + (x/y - \sin y)dy = 0$

• **Assignments for Section 1.10 (The Existance and Uniqueness of ODEs):**

In each of the problems below determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

1. $(t-3)y' + (\ln t)y = 2t, \qquad y(1) = 2$
2. $y' + (\tan t)y = \sin t, \qquad y(\pi) = 0$
3. $(4-t^2)y' + 2ty = 3t^2, \qquad y(1) = -3$

In each of the following problems state where in the ty-plane the hypotheses of Theorem 2 are satisfied.

4. $y' = \dfrac{t-y}{2t+5y}$ 5. $y' = \dfrac{\ln|ty|}{1-t^2+y^2}$ 6. $\dfrac{dy}{dt} = \dfrac{1+t^2}{3y-y^2}$

In each of the following problems solve the given initial value problem and determine how the interval in which the solution exists depends on the initial vale y_0.

7. $y' = -4t/y, \quad y(0) = y_0$ 8. $y' = 2ty^2, \qquad y(0) = y_0$

9. (a) Verify that both $y_1(t) = 1 - t$ and $y_2(t) = -t^2/4$ are solutions of the intial value problem

$$y' = \frac{-t + (t^2 + 4y)^{1/2}}{2}, \qquad y(2) = -1.$$

where are these solutions valid?
(b) Explain why the existence of two solutions of the given problem does not contradict the uniqueness part of Theorem 2.
(c) Show that $y = ct + c^2$ where c is an arbitrary constant, satisfies the differential equation in part (a) for $t \geq -2c$. If $c = -1$, the initial condition is also satisfied, and the solution $y = y_1(t)$ is obtained. Show that there is no choice of c that gives the second solution $y = y_2(t)$.

10. Let $y = y_1(t)$ be a solution of

$$y' + p(t)y = 0, \tag{I}$$

and let $y = y_2(t)$ be a solution of

$$y' + p(t)y = g(t). \tag{II}$$

Show that $y = y_1(t) + y_2(t)$ is also a solution of Eq. (II).