M427J: Differential Equations with Linear Algebra

Homework # 04

Handout: 02/07/2017, Tuesday
Due: 02/15/2017, Wednesday

• Submission: Please make your homework neat and STAPLED. You have to submit your homework Wednesday in the Problem Session. Note that no late homework will be accepted.

• Assignments for Section 2.1: The Solutions to the Linear Homogeneous Equations

In each of the following problems find the Wronskian of the given pair of functions.

1. $x, xe^x$  
2. $\cos^2 \theta, 1 + \cos 2\theta$

In each of the following problems, determine the longest interval in which the given initial value problem is certain to have a unique twice differentiable solution. Do not attempt to find the solution.

3. $(t - 1)y'' - 3ty' + 4y = \sin t, \quad y(-2) = 2, \quad y'(-2) = 1$
4. $(x - 3)y'' + xy' + (\ln |x|)y = 0, \quad y(1) = 0, \quad y'(1) = 1$

5. If the Wronskian $W$ of $f$ and $g$ is $3e^{4t}$, and if $f(t) = e^{2t}$, find $g(t)$.

6. If $W(f, g)$ is the Wronskian of $f$ and $g$, and if $u = 2f - g, v = f + 2g$, find the Wronskian $W(u, v)$ of $u$ and $v$ in terms of $W(f, g)$.

• Assignments for Section 2.2: The Homogeneous Equations with Constant Coefficients (I)

In each of the problems below find the general solution of the given differential equation.

1. $6y'' - y' - y = 0$  
2. $y'' + 5y' = 0$  
3. $y'' - 9y' + 9y = 0$

In each of the following problems find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as $t$ increases.

4. $y'' + y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = 1$
5. $6y'' - 5y' + y = 0, \quad y(0) = 4, \quad y'(0) = 0$
6. $y'' + 5y' + 3y = 0, \quad y(0) = 1, \quad y'(0) = 0$
7. Find a differential equation whose general solution is \( y = c_1 e^{2t} + c_2 e^{-3t} \).

In each of Problems 8 and 9 determine the value of \( \alpha \), if any, for which all solutions tend to zero as \( t \to \infty \); also determine the value of \( \alpha \), if any, for which all (nonzero) solutions become unbounded as \( t \to \infty \).

8. \( y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0 \)

9. \( y'' + (3 - \alpha)y' - 2(\alpha - 1)y = 0 \)