M427J: Differential Equations with Linear Algebra
Homework # 04
Handout: 02/07/2017, Tuesday
Due: 02/15/2017, Wednesday

• **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Wednesday** in the Problem Session. Note that *no late homework will be accepted.*

• **Assignments for Section 2.1: The Solutions to the Linear Homogeneous Equations**

In each of the following problems find the Wronskian of the given pair of functions.

1. $x, \quad xe^x$ 2. $\cos^2\theta, \quad 1+\cos 2\theta$

In each of the following problems, determine the longest interval in which the given initial value problem is certain to have a unique twice differentiable solution. Do not attempt to find the solution.

3. $(t-1)y'' - 3ty' + 4y = \sin t, \qquad y(-2) = 2, \quad y'(-2) = 1$
4. $(x-3)y'' + xy' + (\ln|x|)y = 0, \qquad y(1) = 0, \quad y'(1) = 1$

5. If the Wronskian W of f and g is $3e^{4t}$, and if $f(t) = e^{2t}$, find $g(t)$.

6. If $W(f, g)$ is the Wronskian of f and g, and if $u = 2f - g$, $v = f + 2g$, find the Wronskian $W(u, v)$ of u and v in terms of $W(f, g)$.

• **Assignments for Section 2.2: The Homogeneous Equations with Constant Coefficients (I)**

In each of the problems below find the general solution of the given differential equation.

1. $6y'' - y' - y = 0$ 2. $y'' + 5y' = 0$ 3. $y'' - 9y' + 9y = 0$

In each of the following problems find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

4. $y'' + y' - 2y = 0, \qquad y(0) = 1, \quad y'(0) = 1$
5. $6y'' - 5y' + y = 0, \qquad y(0) = 4, \quad y'(0) = 0$
6. $y'' + 5y' + 3y = 0, \qquad y(0) = 1, \quad y'(0) = 0$

7. Find a differential equation whose general solution is $y = c_1 e^{2t} + c_2 e^{-3t}$.

In each of Problems 8 and 9 determine the value of α, if any, for which all solutions tend to zero as $t \to \infty$; also determine the value of α, if any, for which all (nonzero) solutions become unbounded as $t \to \infty$.

8. $y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0$
9. $y'' + (3 - \alpha)y' - 2(\alpha - 1)y = 0$