M427J: Differential Equations with Linear Algebra
Homework # 05
Handout: 02/14/2017, Tuesday
Due: 03/01/2017, Wednesday

• **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Wednesday** in the Problem Session. Note that *no late homework will be accepted.*

• **Assignments for Section 2.2: The Homogeneous Equations with Constant Coefficients (II)**

In each of the following problems find the general solution of the given differential equation.

1. $y'' - 2y' + 6y = 0$
2. $y'' + 6y' + 13y = 0$
3. $y'' + y' + 1.25y = 0$

10. $y'' + 2y' + 2y = 0$
12. $4y'' + 9y = 0$
16. $y'' + 4y' + 6.25y = 0$

In each of the following problems find the solution of the given differential equation. Sketch the graph of the solution and describe its behavior for increasing t.

4. $y'' - 2y' + 5y = 0, \qquad y(\pi/2) = 0, \quad y'(\pi/2) = 2$
5. $y'' + 2y' + 2y = 0, \qquad y(\pi/4) = 2, \quad y'(\pi/4) = -2$

6. Show that $W(e^{\lambda t}\cos(\mu t), e^{\lambda t}\sin(\mu t)) = \mu e^{2\lambda t}$.

7. Using Euler's formula, show that

$$\cos t = (e^{it} + e^{-it})/2, \qquad \sin t = (e^{it} - e^{-it})/(2i).$$

• **Assignments for Section 2.2: The Homogeneous Equations with Constant Coefficients (III)**

In each of the following problem, find the general solution of the given differential equation.

1. $y'' - 2y' + y = 0$
2. $4y'' - 4y' - 3y = 0$
3. $4y'' + 12y' + 9y = 0$
4. $16y'' + 24y' + 9y = 0$
5. $2y'' + 2y' + y = 0$

6. Consider the initial value problem

$$9y'' + 12y' + 4y = 0, \qquad y(0) = a > 0, \qquad y'(0) = -1.$$

(a) Solve the initial value problem.
(b) Find the critical value of a that separates solutions that become negative from those that are always positive.

In each of the following problem, use the method of reduction of order to find a second solution of the given differential equation.

7. $t^2y'' - 4ty' + 6y = 0, \quad t > 0; \quad y_1(t) = t^2$
8. $t^2y'' + 3ty' + y = 0, \quad t > 0; \quad y_1(t) = t^{-1}$
9. $xy'' - y' + 4x^3y = 0, \quad x > 0; \quad y_1(x) = \sin x^2$

• Assignments for Section 2.5: The Nonhomogeneous Equations – the Method of Undetermined Coefficients

In each of the following problem find the solution of the differential equation.

1. $y'' - 2y' - 3y = -3te^{-t}$
2. $y'' + 9y = t^2e^{3t} + 6$
3. $2y'' + 3y' + y = t^2 + 3\sin t$
4. $u'' + \omega_0^2 u = \cos\omega t, \quad \omega^2 \neq \omega_0^2$
5. $y'' + y' + 4y = 2\sinh t,$ Hint: $\sinh t = (e^t - e^{-t})/2$

In each of the following problem find the solution of the initial value problem.

6. $y'' + y' - 2y = 2t, \quad y(0) = 0, \quad y'(0) = 1$
7. $y'' + 4y = t^2 + 3e^t, \quad y(0) = 0, \quad y'(0) = 2$
8. $y'' - 2y' - 3y = 3te^{2t}, \quad y(0) = 1, \quad y'(0) = 0$
9. $y'' + 2y' + 5y = 4e^{-t}\cos 2t, \quad y(0) = 1, \quad y'(0) = 0$

10. In this problem we indicate an alternative procedure for solving the differential equation

$$y'' + by' + cy = (D^2 + bD + c)y = g(t), \tag{1}$$

where b and c are constants, and D denotes differentiation with respect to t. Let r_1 and r_2 be the zeros of the characteristic polynomial of the corresponding homogeneous equation. These roots may be real and different, real and equal, or conjugate complex numbers.
(a) Verify that Eq. (1) can be written in the factored form

$$(D - r_1)(D - r_2)y = g(t)$$

where $r_1 + r_2 = -b$ and $r_1r_2 = c$.
(b) Let $u = (D - r_2)y$. Then show that the solution of Eq. (1) can be found by solving the following two first order equations:

$$(D - r_1)u = g(t), \qquad (D - r_2)y = u(t).$$

In each of the following problems, use the method of Problem 10 to solve the given differential equation.

11. $y'' - 3y' - 4y = 3e^{2t}$
12. $2y'' + 3y' + y = t^2 + 3\sin t$