M427J: Differential Equations with Linear Algebra
Homework # 06
Handout: 02/28/2017, Tuesday
Due: 03/08/2017, Wednesday

• **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Wednesday** in the Problem Session. Note that *no late homework will be accepted.*

• **Assignments for Section 2.4: The Nonhomogeneous Equations – The Method of Variation of Parameters**

In each of the following problems, use the method of variation of parameters to find a particular solution of the given differential equation. Then check your answer by using the method of undetermined coefficients.

1. $y'' - y' - 2y = 2e^{-t}$ 2. $4y'' - 4y' + y = 16e^{t/2}$

In each of the following problems, find the general solution of the given differential equation.

3. $y'' + 9y = 9\sec^2 3t, \quad 0 < t < \pi/6$ 4. $y'' + 4y = 3\csc 2t, \quad 0 < t < \pi/2$

In each of Problems 14 through 16 verify that the given functions y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneous equation.

5. $t^2y'' - t(t+2)y' + (t+2)y = 2t^3, \quad t > 0; \quad y_1(t) = t, \quad y_2(t) = te^t$
6. $ty'' - (1+t)y' + y = t^2e^{2t}, \quad t > 0; \quad y_1(t) = 1 + t, \quad y_2(t) = e^t$
7. $(1-t)y'' + ty' - y = 2(t-1)^2e^{-t}, \quad 0 < t < 1; \quad y_1(t) = e^t, \quad y_2(t) = t$

• **Assignments for Section 2.8 (I&II): Series Solution Near An Ordinary Point**

In each of the following problem determine the radius of convergence of the given power series.

1. $\sum_{n=0}^{\infty} \frac{n}{2^n} x^n$ 2. $\sum_{n=0}^{\infty} 2^n x^n$ 3. $\sum_{n=1}^{\infty} \frac{(x-x_0)^n}{n}$ 4. $\sum_{n=1}^{\infty} \frac{n!x^n}{n^n}$

In each of the following problems,
(a) Seek power series solutions of the given differential equation about the given point x_0; find the recurrence relation.

(b) Find the first four terms in each of two solutions y_1 and y_2 (unless the series terminates sooner).
(c) By evaluating the Wrongkian $W(y_1, y_2)(x_0)$, show that y_1 and y_2 form a fundamental set of solutions.
(d) If possible, find the general term in each solution.

5. $y'' - xy' - y = 0, \quad x_0 = 1$
6. $(1 - x)y'' + y = 0, \quad x_0 = 0$
7. $y'' + xy' + 2y = 0, \quad x_0 = 0$
8. $(1 + x^2)y'' - 4xy' + 6y = 0, \quad x_0 = 0.$

In each of the following problems, determine a lower bound for the radius of convergence of series solutions about each given point x_0 for the given differential equation.

9. $y'' + 4y' + 6xy = 0; \quad x_0 = 0, \quad x_0 = 4.$
10. $(x^2 - 2x - 3)y'' + xy' + 4y = 0; \quad x_0 = 4, \quad x_0 = -4, \quad x_0 = 0.$
11. $(1 + x^3)y'' + 4xy' + y = 0; \quad x_0 = 0, \quad x_0 = 2.$
12. $xy'' + y = 0; \quad x_0 = 1.$