M427J: Differential Equations with Linear Algebra
Homework # 08
Handout: 03/21/2017, Tuesday
Due: 04/05/2017, Wednesday

• **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Wednesday** in the Problem Session. Note that *no late homework will be accepted.*

• **Assignments for Section 3.1: Matrices and Vectors**

1. If

$$\mathbf{A} = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 0 & 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 2 & 1 & -1 \\ -2 & 3 & 3 \\ 1 & 0 & 2 \end{pmatrix}, \qquad \text{and } \mathbf{C} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 2 \\ 0 & 1 & -1 \end{pmatrix},$$

verify that
(a) $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$
(b) $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$
(c) $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$

2. If

$$\mathbf{x} = \begin{pmatrix} 2 \\ 3i \\ 1 - i \end{pmatrix} \text{ and } \mathbf{y} = \begin{pmatrix} -1 + i \\ 2 \\ 3 - i \end{pmatrix},$$

find
(a) $\mathbf{x}^T\mathbf{y}$
(b) $\mathbf{y}^T\mathbf{y}$
(c) $(\mathbf{x}, \mathbf{y})$
(d) $(\mathbf{y}, \mathbf{y})$

3. If

$$\mathbf{A}(t) = \begin{pmatrix} e^t & 2e^{-t} & e^{2t} \\ 2e^t & e^{-t} & -e^{2t} \\ -e^t & 3e^{-t} & 2e^{2t} \end{pmatrix}, \qquad \text{and } \mathbf{B}(t) = \begin{pmatrix} 2e^t & e^{-t} & 3e^{2t} \\ -e^t & 2e^{-t} & e^{2t} \\ 3e^t & -e^{-t} & -e^{2t} \end{pmatrix},$$

find
(a) $\mathbf{A} + 3\mathbf{B}$
(b) $\mathbf{AB}$
(c) $d\mathbf{A}/dt$

In each of Problems 4 and 5 verify that the given matrix satisfies the given differential equation.

$$4.\ \boldsymbol{\Psi}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \boldsymbol{\Psi}, \qquad \boldsymbol{\Psi}(t) = \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix}.$$

$$5.\ \boldsymbol{\Psi}' = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \boldsymbol{\Psi}, \qquad \boldsymbol{\Psi}(t) = \begin{pmatrix} e^{t} & e^{-2t} & e^{3t} \\ -4e^{t} & -e^{-2t} & 2e^{3t} \\ -e^{t} & -e^{-2t} & e^{3t} \end{pmatrix}.$$

- **Assignments for Section 3.2: Linear Dependence of Vectors; Determinant**

In each of the following problem determine whether the members of the given set of vectors are linearly independent. If they are linearly dependent, find a linear relation among them.

1. $\mathbf{x}^{(1)} = (1,1,0)$, $\quad \mathbf{x}^{(2)} = (0,1,1)$, $\quad \mathbf{x}^{(3)} = (1,0,1)$.
2. $\mathbf{x}^{(1)} = (2,1,0)$, $\quad \mathbf{x}^{(2)} = (0,1,0)$, $\quad \mathbf{x}^{(3)} = (-1,2,0)$.
3. $\mathbf{x}^{(1)} = (1,2,2,3)$, $\quad \mathbf{x}^{(2)} = (-1,0,3,1)$, $\quad \mathbf{x}^{(3)} = (-2,-1,1,0)$ $\quad \mathbf{x}^{(4)} = (-3,0,-1,3)$.

For each of the following matrix, compute the determinant.

$$4.\ \begin{pmatrix} 2 & -1 & 3 \\ 2 & 3 & 1 \\ 1 & 0 & -1 \end{pmatrix} \qquad 5.\ \begin{pmatrix} 1 & 3 & -2 \\ 4 & 0 & -1 \\ -1 & 1 & 1 \end{pmatrix} \qquad 6.\ \begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 3 \\ 0 & -2 & 1 \end{pmatrix}$$

- **Assignments for Section 3.3: System of Linear Equations**

In each of the following problems transform the given equation into a system of first order equations.

$$1.\ u'' + 0.5u' + 2u = 3\sin t \qquad 2.\ u^{(4)} - u = 0$$

3. Consider the linear homogeneous system

$$x' = p_{11}(t)x + p_{12}(t)y,$$
$$y' = p_{21}(t)x + p_{22}(t)y.$$

Show that if $x = x_1(t),\ y = y_1(t)$ and $x = x_2(t),\ y = y_2(t)$ are two solutions of the given system, then $x = c_1x_1(t) + c_2x_2(t)$, $y = c_1y_1(t) + c_2y_2(t)$ is also a solution for any constants c_1 and c_2. This is the principle of superposition.

4. Let $x = x_1(t),\ y = y_1(t)$ and $x = x_2(t),\ y = y_2(t)$ be any two solutions of the linear nonhomogeneous system

$$x' = p_{11}(t)x + p_{12}(t)y + g_1(t),$$
$$y' = p_{21}(t)x + p_{22}(t)y + g_2(t).$$

Show that $x = x_1(t) - x_2(t)$, $y = y_1(t) - y_2(t)$ is a solution of the corresponding homogeneous system.