M427J: Differential Equations with Linear Algebra
Homework # 11
Handout: 04/18/2017, Tuesday
Due: 04/26/2017, Wednesday

• **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Monday** in the Problem Session. Note that *no late homework will be accepted.*

• **Assignments for Section 5.1 (II): Eigenvalue Problems**

In each of the following problems, find the eigenvalues and eigenfunctions of the given boundary value problem. Assume that all eigenvalues are real.

1. $y'' + \lambda y = 0, \quad y(0) = 0, \quad y'(\pi) = 0$
2. $y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(\pi) = 0$
3. $y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(\pi) = 0$
4. $y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(L) = 0$

• **Assignments for Section 5.2: Fourier Series**

In each of the following problems, determine whether the given function is periodic. If so, find its fundamental period.

1. $\cos 2\pi x$ 2. $\sin \pi x/L$ 3. $f(x) = \begin{cases} 0, & 2n-1 \le x < 2n \\ 4, & 2n \le x < 2n+1 \end{cases} \qquad n = 0, \pm 1, \pm 2, ...$

In each of Problem 4 through 6:
(a) Sketch the graph of the given function for three periods.
(b) Find the Fourier series for the given function.

4. $f(x) = -x, \quad -L \le x < L; \qquad f(x+2L) = f(x)$

5. $f(x) = \begin{cases} x, & -\pi \le x < 0 \\ 0, & 0 \le x < \pi; \end{cases} \qquad f(x+2\pi) = f(x)$

6. $f(x) = \begin{cases} x+1, & -1 \le x < 0 \\ 1-x, & 0 \le x < 1; \end{cases} \qquad f(x+2) = f(x)$

In each of Problem 7 through 9:
(a) Find the Fourier series for the given function.
(b) Sketch the graph of the Fourier series (where they converge) for three periods.

7. $f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ x, & 0 \le x < \pi \end{cases}$ 8. $f(x) = 1 - x^2, \quad -1 \le x < 1$ 9. $f(x) = \begin{cases} 0, & -1 \le x < 0 \\ x^2, & 0 \le x < 1 \end{cases}$

10. Find the solution of the initial value problem

$$y'' + \omega^2 y = \sin nt, \qquad y(0) = 0, \qquad y'(0) = 0,$$

where n is a positive integer and $\omega^2 \neq n^2$. What happens if $\omega^2 = n^2$?