M427J: Differential Equations with Linear Algebra
Homework # 12
Handout: 04/25/2017, Tuesday
Due: 05/03/2017, Wednesday

• **Submission:** Please make your homework neat and **STAPLED**. You have to submit your homework **Wednesday** in the Problem Session. Note that *no late homework will be accepted.*

• **Assignments for Section 5.3: The Even and Odd Functions**

In each of the following problems determine whether the given function is even, odd, or neither.

1. $x^3 - 2x + 1$ 2. $\tan 2x$

In each of the following problems a function f is given on an interval of length L. In each case sketch the graphs of the even and odd extensions of f of period $2L$.

3. $f(x) = \begin{cases} 0, & 0 \le x < 1 \\ x - 1, & 1 \le x < 2 \end{cases}$

4. $f(x) = x - 3, \quad 0 < x < 4$

In each of the following problems find the required Fourier series for the given function, and sketch the graph of the function to which the series converges over three periods.

5. $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & 1 < x < 2; \end{cases}$ cosine series, period 4

6. $f(x) = \begin{cases} x, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$ sine series, period 4

• **Assignments for Section 5.4: Separition of Varibles**

In each of the following problems, determine whether the method of separation of variables can be used to replace the given partial differential equation by a pair of ordinary differential equations. If so, find the equations.

1. $tu_{xx} + xu_t = 0$ 2. $\left[p(x)u_x\right]_x - r(x)u_{tt} = 0$ 3. $u_{xx} + u_{yy} + xu = 0$

4. Find the solution of the heat conduction problem

$$\begin{aligned} &100u_{xx} = u_t, \qquad 0 < x < 1, \quad t > 0; \\ &u(0,t) = 0, \qquad u(1,t) = 0, \quad t > 0; \\ &u(x,0) = \sin 2\pi x - \sin 5\pi x, \qquad 0 \le x \le 1. \end{aligned}$$

5. Find the solution of the heat conduction problem

$$u_{xx} = 4u_t, \qquad 0 < x < 2, \quad t > 0;$$
$$u(0,t) = 0, \qquad u(2,t) = 0, \quad t > 0;$$
$$u(x,0) = 2\sin(\pi x/2) - \sin \pi x + 4 \sin 2\pi x, \qquad 0 \le x \le 2.$$

6. Find the solution $u(x, y)$ of Laplace's equation in the rectangle $0 < x < a$, $0 < y < b$, that satisfies the boundary conditions

$$u(0,y) = 0, \qquad u(a,y) = 0, \qquad 0 < y < b,$$
$$u(x,0) = h(x), \qquad u(x,b) = 0, \qquad 0 \le x \le a.$$