Guidelines for Writing Elementa Proofs in
SET THEORY AND OTHER NOTES

Definition: A set \( A \) is non-empty or \( A \neq \emptyset \)
\[ \iff \text{There exists an element } x \in U \text{ such that } x \in A. \]

Guidelines for Writing Elementa Proofs

1. \[ \therefore x \in A \land B \quad [\text{This is followed by:}] \]
   \[ \therefore x \in A \land x \in B \text{ by definition of "Intersection"} \]

2. \[ \therefore x \in A \lor B \quad [\text{This is followed by:}] \]
   \[ \therefore x \in A \lor x \in B \text{ by definition of "Union"} \]
   \[ \begin{align*}
   \text{CASE 1 (} x \in A) \\
   \therefore \quad \because \text{ in CASE 1}. \\
   \text{CASE 2 (} x \in B) \\
   \therefore \quad \because \text{ in CASE 2}. 
   \end{align*} \]

3. \[ \therefore C \cap D \neq \emptyset \quad [\text{is followed by:}] \]
   \[ \therefore \text{There exists an element } x \in U \text{ such that } x \in C \cap D. \quad [\text{See 1 Next}] \]

4. Prove "\( A \cap B = \emptyset \)" using a proof-by-contradiction.
Guidelines (Continued)

5. To prove "$x \in A \cup B$":
   \[ \begin{aligned}
   &\text{[First prove } x \in A] \quad \text{or} \quad [\text{First prove } x \in B] \\
   &\quad \therefore x \in A \\
   &\quad x \in A \text{ or } x \in B \text{ by Generalization} \\
   &\quad x \in A \cup B \text{ by definition of "union"}
   \end{aligned} \]

6. To prove "$x \in A \cap B$"
   \[ \begin{aligned}
   &\text{[First, prove } x \in A] \quad \therefore x \in A \\
   &\text{[Next, prove } x \in B] \quad \therefore x \in B \\
   &\quad \therefore x \in A \text{ and } x \in B \text{ by Conjunction} \\
   &\quad x \in A \cap B \text{ by definition of "intersection"}
   \end{aligned} \]

7. To prove "$x \in A - B$"
   \[ \begin{aligned}
   &\text{[First, prove } x \in A] \quad \therefore x \in A \\
   &\text{[Next, prove } x \in B] \quad \therefore x \in B \\
   &\quad \therefore x \in A \text{ and } x \notin B \text{ by Conjunction} \\
   &\quad x \in A - B \text{ by definition of set difference}
   \end{aligned} \]

8. $x \in C$ and $x \in D$ \[ \text{[is followed by:]} \]
   \[ \begin{aligned}
   &\therefore x \in C \text{ by Specialization. [That is, if you need this fact]} \\
   &\therefore x \in D \text{ by Specialization [That is, if you need this fact]}
   \end{aligned} \]

9. $x \in C$ \[ \text{[is followed by:]} \]
   \[ \begin{aligned}
   &\therefore x \in C \text{ by definition of "complement"}
   \end{aligned} \]

10. $x \in A \cap B$ \[ \text{[is followed by:]} \]
    \[ \begin{aligned}
    &\text{It is false that } x \in A \cap B. \\
    &\text{It is false that } x \in A \text{ and } x \in B \text{ by definition of "\cap".} \\
    &x \in A \text{ or } x \in B \text{ by De Morgan's Laws of Logic.}
    \end{aligned} \]
Proof Fragments showing the applications of universal modus ponens and universal modus tollens in writing elemental proofs in set theory:

**Proof Fragment**
Suppose $A \subseteq B$.

Suppose $t \in A$.

$. . . t \in B$ by universal modus ponens.

**Definition**
For all $x \in U$, if $x \in A$, then $x \in B$.

$t \in U$ and $t \in A$.

$. . . t \in B$ by universal modus ponens.

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**Proof Fragment**
Suppose $A \subseteq B$.

Suppose $t \in B$.

$. . . t \in A$ by universal modus tollens.

**Definition**
For all $x \in U$, if $x \in A$, then $x \in B$.

$t \in U$ and $t \in A$.

$. . . t \in A$ by universal modus tollens.