A Proof Design for Proofs of "A \subseteq B"

Let A and B be sets.

The definition of "A \subseteq B" has two equivalent forms:

**Defn 1:**

\[ A \subseteq B \iff \forall x \in A, \text{if } x \in A, \text{ then } x \in B. \]

**Defn 2:**

\[ A \subseteq B \iff \forall x \in A, \ x \in B. \]

These are equivalent because the actual domain of x in Defn 2 is the same set A as the effective domain of x in Defn 1.

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**To Prove:** A \subseteq B

**Proof:** Let x \in A be given.

[N.T.S.: x \in B]

\[ \vdash x \in B. \]

\[ \vdash A \subseteq B, \text{ by Direct Proof.} \]

QED

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**Example:**

**To Prove:** For all sets A and B, (A \cap B) \subseteq A.

**Proof:**

Let A and B be any sets.

[N.T.S.: (A \cap B) \subseteq A.]

Let x \in A \cap B be given.

\[ \vdash x \in A \text{ AND } x \in B, \text{ by def of "intersection".} \]

\[ \vdash x \in A, \text{ by Specialization.} \]

\[ \vdash \forall x \in A \cap B, x \in A \]

\[ \vdash (A \cap B) \subseteq A, \text{ by Direct Proof.} \]

QED