AN EXAMPLE MODEL PROOF TO HELP WITH HW # 5A, PART II

TO PROVE: FOR ALL INTEGERS $k \geq 4$,

if $1+2+\cdots+k = \frac{k(k+1)}{2}$,

then $1+2+\cdots+(k+1) = \frac{(k+1)(k+2)}{2}$.

Proof: Let k be any integer such that $k \geq 4$.

Suppose that $1+2+\cdots+k = \frac{k(k+1)}{2}$.

[WE NTS: $1+2+3+\cdots+(k+1) = \frac{(k+1)(k+2)}{2}$.]

Consider the fact that

$$1+2+\cdots+k+(k+1) = (1+2+\cdots+k)+k+1.$$

By substitution, using the equality in the supposition above,

$$\begin{aligned} 1+2+3+\cdots+k+(k+1) &= \frac{k(k+1)}{2} + (k+1) \\ &= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} \\ &= \frac{(k^2+k)+2k+2}{2} \\ &= \frac{k^2+3k+2}{2}. \end{aligned}$$

$\therefore$ $1+2+3+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$.

$\therefore$ For all integers $k \geq 4$,

if $1+2+\cdots+k = \frac{k(k+1)}{2}$,

then $1+2+\cdots+k+(k+1) = \frac{(k+1)(k+2)}{2}$,

by Direct Proof.

Q.E.D.