M 340 L - Quiz 2

06-12-08

1 Exercise 1.7, # 1 (solution)

Let

$$\mathbf{u} := \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{v} := \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix} \qquad \mathbf{w} := \begin{bmatrix} 9 \\ 4 \\ 4 \end{bmatrix}$$

We want to know if there is a non trivial solution to

$$x_1\mathbf{u} + x_2\mathbf{v} + x_3\mathbf{w} = 0 \tag{1}$$

We work on the augmented matrix:

$$\begin{pmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -6 & -8 & 0 \end{pmatrix} \quad \underset{R_3+3R_2}{\longrightarrow} \quad \begin{pmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 4 & 0 \end{pmatrix}$$

There are no free variables, hence (1) has a unique solution (namely the trivial one). Therefore, $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly independent.