

M 340 L - Quiz 5

06-26-08

1 Exercise 4.1, # 21

Is this a subspace of $M_{2 \times 2}$?

$$H := \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \in M_{2 \times 2} \right\}$$

1.1 Solution 1

$$\begin{aligned} H &= \left\{ a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} : a, b, c \in \mathbb{R} \right\} \\ &= \text{Span} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \end{aligned}$$

H is the span of some vectors, therefore it is a subspace.

1.2 Solution 2

Check subspace axioms.

1. $0 \in H$
True, with $a = b = c = 0$.
2. closed under +

$$\begin{bmatrix} a_1 & b_1 \\ 0 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ 0 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ 0 + 0 & d_1 + d_2 \end{bmatrix} \in M_{2 \times 2}$$

3. closed under scalar multiplication

For any scalar λ :

$$\lambda \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = \begin{bmatrix} \lambda a & \lambda b \\ 0 & \lambda d \end{bmatrix} \in M_{2 \times 2}$$

Therefore H is indeed a subspace of $M_{2 \times 2}$.