4.6 #3

3. The matrix B is in echelon form. There are three pivot columns, so the dimension of Col A is 3. There are three pivot rows, so the dimension of Row A is 3. There are two columns without pivots, so the equation $A\mathbf{x} = \mathbf{0}$ has two free variables. Thus the dimension of Nul A is 2. A basis for Col A is the pivot columns of A:

$$\left\{ \begin{bmatrix} 2 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \\ -4 \end{bmatrix} \right\}.$$

A basis for Row A is the pivot rows of B: $\{(2,-3,6,2,5),(0,0,3,-1,1),(0,0,0,1,3)\}$. To find a basis for Nul A row reduce to reduced echelon form:

$$A \sim \begin{bmatrix} 1 & -3/2 & 0 & 0 & -9/2 \\ 0 & 0 & 1 & 0 & 4/3 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The solution to $A\mathbf{x} = \mathbf{0}$ in terms of free variables is $x_1 = (3/2)x_2 + (9/2)x_5$, $x_3 = -(4/3)x_5$, $x_4 = -3x_5$, with x_2 and x_5 free. Thus a basis for Nul A is

$$\left\{ \begin{bmatrix} 3/2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 9/2 \\ 0 \\ -4/3 \\ -3 \\ 1 \end{bmatrix} \right\}.$$