

6.3 # 11

11. Note that  $\mathbf{v}_1$  and  $\mathbf{v}_2$  are orthogonal. The Best Approximation Theorem says that  $\hat{\mathbf{y}}$ , which is the orthogonal projection of  $\mathbf{y}$  onto  $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$ , is the closest point to  $\mathbf{y}$  in  $W$ . This vector is

$$\hat{\mathbf{y}} = \frac{\mathbf{y} \cdot \mathbf{v}_1}{\mathbf{v}_1 \cdot \mathbf{v}_1} \mathbf{v}_1 + \frac{\mathbf{y} \cdot \mathbf{v}_2}{\mathbf{v}_2 \cdot \mathbf{v}_2} \mathbf{v}_2 = \frac{1}{2} \mathbf{v}_1 + \frac{3}{2} \mathbf{v}_2 = \begin{bmatrix} 3 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$