In Part A, do you think this sum may be related to the genus of the 2-manifold? Why?

Generalization

If you had to guess, do you think the sum of the indices is independent of the choice of vector field on $S^2$? Why or why not?

If the sum of the indices was independent of the choice of vector field, then $\chi(S^2)$, which is equal to the Euler Characteristic of $S^2$, would be zero.

Do you think there is a vector field on $S^2$ with exactly one non-trivial zero? Why or why not?

In Part B, let $X$ be a smooth vector field with exactly $n$ non-trivial zeros.

Why does this relate to the hairy ball theorem?

Why or why not?

Is this value consistent with the existence of a smooth vector field on $S^2$ with no zeros?

What does this hint tell you about the sum of the indices?

Can you find a vector field on $S^2$ with no zeros?

Explain why or why not.

Suppose the sum of the indices was independent of the choice of the vector field on $S^2$.

If you had a vector field on $S^2$ with exactly $n$ zeros, would you expect the sum of the indices of these zeros to be $-2n$?

If you had a vector field on $S^2$ with exactly three zeros, would you expect the sum of these zeros to be $-6$?

Can you find a vector field on $S^2$ with exactly one zero?

What is the index of this zero?

What is the sum of the indices of all the zeros of this vector field?

What is the index of each of these zeros?

Can you find a smooth vector field on $S^2$ with exactly two zeros?

Investigation: On which closed 2-manifolds and for which positive integers $n$, does there exist a smooth vector field with exactly $n$ non-trivial zeros?