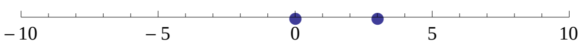
Worksheet

Math Circle September 11, 2011

Problem 1 *How far apart are the following pairs of points*



a

b

c In general how do we tell how far two points are on the real line?

Problem 2

a Which points on the real line are $1-$*close to 0? Try to draw a picture of them*

b If $r > 0$*, which points on the real line are* $r-$*close to 0?*

c If $r > 0$ *and* x *is a real number, which points on the real line are* $r-$*close to* x*?*

Problem 3 *Try to find a pattern for the following sequences.*

a $\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right)$

b $(1, 1, 2, 3, 5, 8, 13, \ldots)$

Problem 4 *If we look at the sequence* $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$

a *How many points do we need to throw away before the rest are* $.23-$*close to 0?*

b *How many points do we need to throw away before the rest are* $.05-$*close to 0?*

c *If* ε *is a very small positive number, how many points do we need to throw away before the rest are* $\varepsilon-$*close to 0? [Hint: Your answer will depend on* ε*.]*

Problem 5

a *Make a graph of the sequence* $\{n\}_{n=1}^{\infty}$ *for the first several values.*

b *Based on this graph, does it appear that the sequence* n *approaches any finite number? If so, what number? If not, why?*

Problem 6

a *Fill in the following tables*

n	$\left\{\left(\frac{1}{2}\right)^n\right\}$
1	.5
2	.25
3	
4	
5	
10	
100	
1000	

n	$\left\{\left(\frac{3}{2}\right)^n\right\}$
1	1.5
2	2.25
3	
4	
5	
10	
100	
1000	

b *Does* $\left\{\left(\frac{1}{2}\right)^n\right\}_{n=1}^{\infty}$ *approach a finite number? If so, what is it?*

c *Does* $\left\{\left(\frac{3}{2}\right)^n\right\}_{n=1}^{\infty}$ *approach a finite number? If so, what is it?*

Problem 7 *For the following sequences write down the sequence of partial sums and try to decide if it approaches a finite number.*

a $(1, -1, 1, -1, 1, -1, \ldots)$

b $\left(1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\right)$

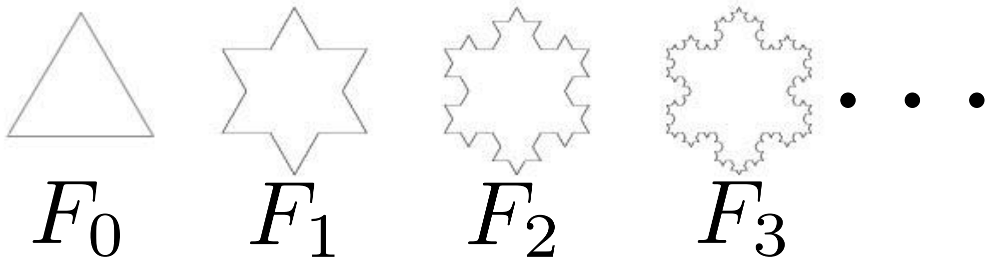
Problem 8

1. *How far can you push the top card before it falls off?*
2. *How far can you push the second card before the first two fall?*
3. *If you had an infinite deck of cards how much or an overhang could you get?*

Try to fill out the following table about the card stacking problem in order to answer the questions above.

# of Cards	Distance You Can Push	Total Overhang
1	1/2	1/2
2	1/4	1/2+1/4
3		
4		
5		
6		
7		
n		

Problem 9



a *If F_0 is an equilateral triangle with side length 1, then what is the area of F_0?*

b *F_1 is created from F_0 by gluing on 3 equilateral triangles of side length $1/3$. What is the area of an equilateral triangle of side length $1/3$? What is the area of F_1?*

c *How many triangles do we have to glue to F_1 to get F_2? How many do we have to glue to F_2 to get F_3?*

d *What is the area of F_2 and F_3? What about F_n?*

e *What is the perimeter of F_1? What about F_2?*

n	# of Δs Added	Area of Δs Added	Area of F_n
1			
2			
3			
4			
5			

n	Side Length	# of Sides	Length of Perim
1	1/3	12	12/3
2			
3			
4			
5			

f *After filling out the tables can you write down a formula for the area of F_n? Can you write down a formula for the perimeter of F_n?*