USING RANDOMNESS TO CATCH CHEATERS

ANDREW J. BLUMBERG

1. Overview

There are three major questions for today:

(1) What is randomness? (How do I know it when I see it?)
(2) What is it good for?
(3) How can I get some?

We're going to develop ideas about randomness in terms of games (the fancy name for what we're talking about is "interactive protocols").

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2. Flipping coins

Game #1:
Each round:

(1) Player 1 guesses either "heads" or "tails".
(2) Player 2 flips a coin.

Scoring:

- If player 1 guessed right, player 1 gets a point.
- If player 1 guessed wrong, player 2 gets a point.

Round	1	2	3	4	5	6	7	8	9	10
Player 1										
Player 2										

Player 1 score: Player 2 score:

Game #2:
Each round:

(1) Player 1 flips a coin to determine a guess of either "heads" or "tails".
(2) Player 2 flips a coin.

Scoring:

- If player 1 guessed right, player 1 gets a point.
- If player 1 guessed wrong, player 2 gets a point.

Round	1	2	3	4	5	6	7	8	9	10
Player 1										
Player 2										

Player 1 score: Player 2 score:

Game #3:
Each round:

(1) Player 1 guesses either "heads" or "tails".
(2) Player 2 guesses either "heads" or "tails".

Scoring:

- If player 1 and player 2 picked the same thing, player 1 gets a point.
- If player 1 and player 2 picked different things, player 2 gets a point.

You can play this using fingers (a lesser "rock-paper-scissors").
(One finger is "heads", two fingers is "tails".)

Round	1	2	3	4	5	6	7	8	9	10
Player 1										
Player 2										

Player 1 score: Player 2 score:

Game #4:

(1) Player 1 secretly picks a pattern of three coin flips: for example, HHT or THT or TTT.
(2) Then, play game #2, except that player 1 has to cycle through the pattern as the turns continue.

For example, if the pattern is HTH, on round 1, player 1 chooses H. On round 2, T. On round 3, H. On round 4, H. On round 5, T. And so on. . .

Round	1	2	3	4	5	6	7	8	9	10
Player 1										
Player 2										

Player 1 score: Player 2 score:

3. Using coin flips to detect cheating

Suppose Bob claims that he can infallibly tell Coke from Pepsi by smell.

You can test him by playing a game.

Each round:

(1) Flip a coin.
(2)
- If the coin is heads, put a cup of Coke on the left, and a cup of Pepsi on the right.
- If the coin is tails, put a cup of Pepsi on the left, and a cup of Coke on the right.

(3) Ask Bob which is which.

Round	1	2	3	4	5	6	7	8	9	10
Coin flip										
Soda sniffer										

Soda sniffer score:

Repeat, now letting Bob taste the soda.

Round	1	2	3	4	5	6	7	8	9	10
Coin flip										
Soda taster										

Soda taster score:

Exercise: You can do this even if you don't know the right answer — imagine that I give you two cups, one with Coke, and one with Pepsi, and don't tell you which is which. You can still test Bob! How?

Exercise: I claim that I know the number of hairs on your head. Design a game to test me.

Exercise: You can do this for lots of things — make up some examples. (For instance, stock picking.)

4. Getting randomness

We think of coins and dice as random.

Exercise: what are some other sources of random numbers? (How can you tell that the numbers are "really random"?)

Suppose you have coins but you want to make a die. How can you do it?

Exercise: Develop a procedure. Test it:

(1) Use your procedure to generate 20 "die rolls". Record the outcomes.

(2) Roll an actual die 20 times. Record the outcomes.

Question: how can we compare the tables?

(Hint: count the number of 1's, 2's, 3's, etc.)

Value	1	2	3	4	5	6
Your procedure						
Real die						

Exercise: What happens if you roll multiple dice and add them? (Make tables!)