Activity 1 — Basic Sets

Here are some questions to get us all up to speed with sets, and the notation that we use to describe them.

1. Suppose P is the set of all prime numbers, $\mathbb{Z}_{odd}$ is the set of all odd integers, and $S = \{5, 13, 17, 303\}$. True or false?

$$P \subset \mathbb{Z}_{odd}$$

$$\mathbb{Z}_{odd} \subset S$$

$$S \subset \mathbb{Z}_{odd}$$

$$S \subset P$$

$$S \subset \mathbb{R}$$

$$P \subset \mathbb{Q}$$

2. Suppose S_p is the set of all words that can be created from any particular phrase p. Here are six different phrases: dormitory, dirty room, dirt, tidy, it, moody. Order the sets S_p by inclusion.

3. A **power set** $\mathscr{P}(S)$ of a set S is the set of all subsets of S. What is the power set $\mathscr{P}(S)$ of $S = \{a, b, c\}$?

4. Describe what the sets are using a regular sentence. Then show what S is with a picture. Is S finite or infinite?

$$S_1 = \{x \in \mathbb{Z} \mid -2 < x < 5\}$$

$$S_2 = \{(x, y) \in \mathbb{R}^2 \mid x \geq 5\}$$

$$S_3 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$$

Activity 2 — Venn Diagrams and Set Properties

1. Illustrate the expressions by drawing Venn diagrams.

   a) $A \cap B^c$

   b) $(A \cup B)^c$

   c) $A^c \cap B^c$

2. Illustrate the expressions by drawing Venn diagrams.

   a) $A \cap B = \emptyset$, $A \subseteq C$ and $C \cap B \neq \emptyset$.

   b) $A \cap B = \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subseteq B$ and $C \not\subseteq B$.

3. Take the universal set to be $\mathbb{R}$, all real numbers, and let A and B be the sets

   $$A = \{x \in \mathbb{R} \mid 0 < x \leq 2\}$$

   $$B = \{x \in \mathbb{R} \mid 1 \leq x \leq 4\}$$

   Determine the following sets:

   a) A^c

   b) B^c

   c) $A^c \cap B^c$

   d) $A^c \cup B^c$

   e) $(A \cap B)^c$

   f) $(A \cup B)^c$

4. **Challenge Problem!** Use an element argument to prove De Morgan's Laws.
   If you get to this problem and don't know what an element argument means or what De Morgan's Law is, ask a Math Circle helper!