Additional Problems for Saturday Morning Math Group

April 27, 2013

- 1. A tropical island near Bora Bora has 60 inhabitants. Each has blue eyes, but somehow doesn't realize it. That's a good thing since local custom demands that on the day a person finds out he/she has blue eyes, then he/she must evacuate the island at midnight. Another local custom is the daily noon lunch gathering, where all of the inhabitants mingle and see each other. Now one beautiful May 1 a stranger arrives on the island and, at the noon gathering, announces "Someone on this island has blue eyes". For the first time each inhabitant begins to think seriously about the possibility that his/her own eyes are blue. What happens?
- 2. A certain city has 10 bus routes. Is it possible to arrange the routes and the bus stops so that if one route is closed, it is still possible to get from any one stop to any other (possibly changing along the way), but if any two routes are closed, there are at least two stops such that it is impossible to get from one to the other?
- 3. A map is obtained by drawing 15 straight lines in the plane. Can the map necessarily be colored using two colors in such a way that no two neighboring countries have the same color?
- 4. In how many ways can you change one dollar? (Each "change" is an exchange of a dollar bill for a certain number of pennies, nickels, dimes, quarters, and half dollars.)
- 5. Among 12 similar coins there is one counterfeit, which may be lighter or heavier than the 11 identical genuine coins. You are allowed 3 weighings on a pan balance, such as pictured below. Can the counterfeit coin be identified, and can you tell if it is lighter or heavier than the others?

