The Idea of a Surface
Some Definitions and Examples
Why do we care?!
Building Surfaces
Pants Decompositions

How a Surface Wears Its Pants

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The University of Texas at Austin

September 21, 2013

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The Idea of a Surface

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The Idea of a Surface

- Let's go back 2000 years!

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The Idea of a Surface

- Let's go back 2000 years!
- What's the shape of Earth?

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- Let's go back 2000 years!
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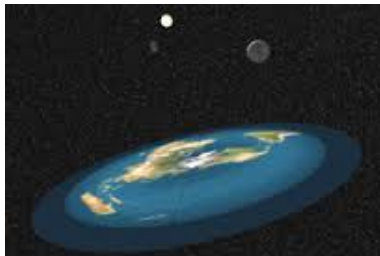
Figure : Flat Earth

- Were they right?

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- Were they right?
- The Earth is ...

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- Were they right?
- The Earth is ... round!

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- Were they right?
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- What was the problem?

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- Were they right?
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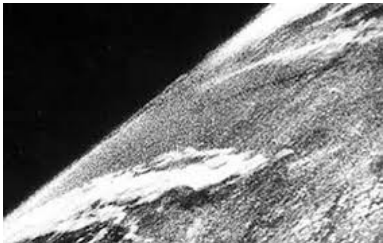- What is the best proof that you have that the Earth is round?

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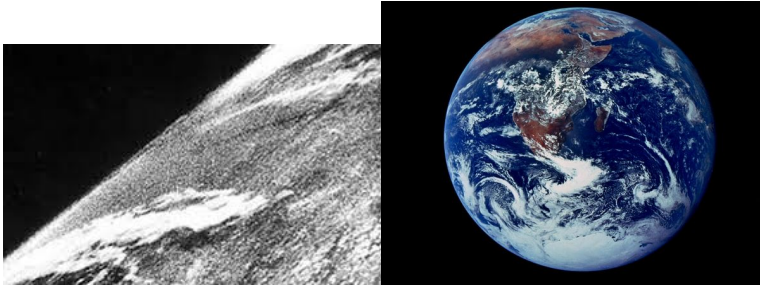Building Surfaces
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- Were they right?
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Figure : First shots of a round Earth

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- There's a huge difference between seeing only a small piece of the earth at once and seeing the WHOLE earth.

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- There's a huge difference between seeing only a small piece of the earth at once and seeing the WHOLE earth.
- Idea: small pieces of "flatness" can patch together to give you very different shapes.

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- There's a huge difference between seeing only a small piece of the earth at once and seeing the WHOLE earth.
- Idea: small pieces of "flatness" can patch together to give you very different shapes.
- The goal of this talk is to discuss this deep fact mathematically.

Definitions

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Definitions

Definition

Flat means that it looks like the *plane* or an "infinite" sheet of paper.

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Definitions

Definition

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Figure : Flatness

The Idea of a Surface
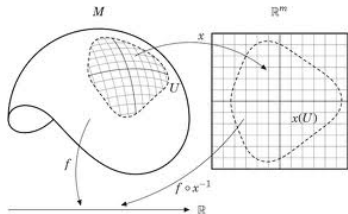**Some Definitions and Examples**
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Definition

A *Surface* is a shape with the following property: at every point of the shape there is an area around it that looks exactly like the area around any point in the plane.

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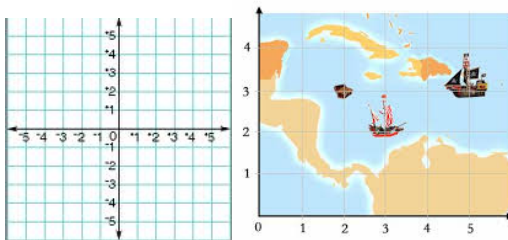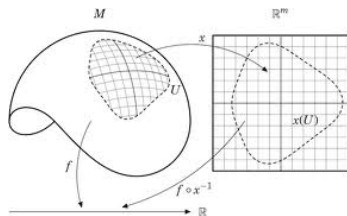
Definition

A *Surface* is a shape with the following property: at every point of the shape there is an area around it that looks exactly like the area around any point in the plane.

Test: Shrink yourself down (very small) and imagine standing at any point on the shape. If you only see flatness (if you think you're in the plane) then the shape is a surface.

Examples

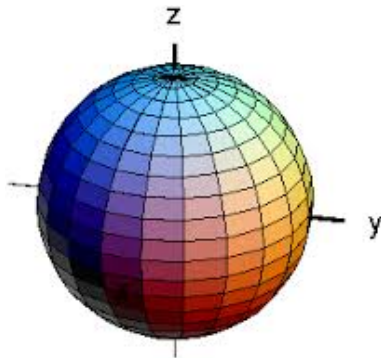The Idea of a Surface
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Examples

Figure : Sphere

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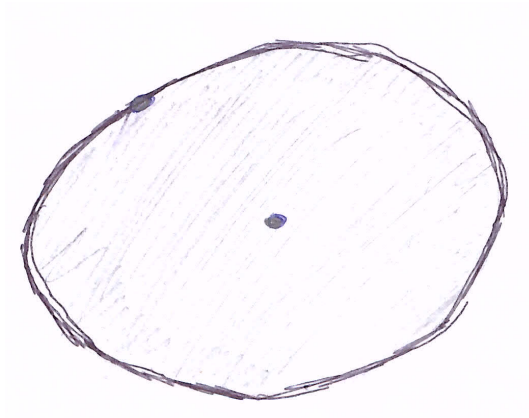Why do we care?!
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Figure : Disk

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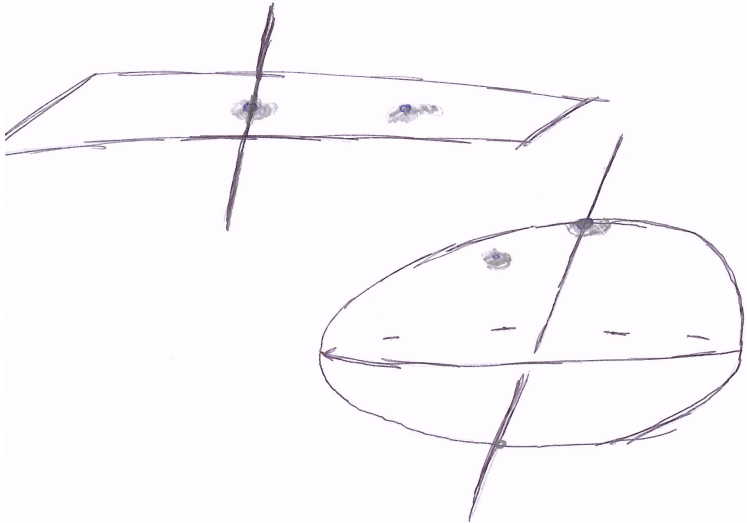
Activity

- First draw 3 examples of shapes that are NOT surfaces.
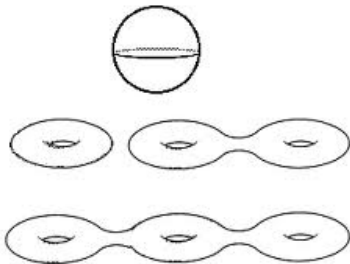- Then draw a surface that is NOT the plane or the sphere.

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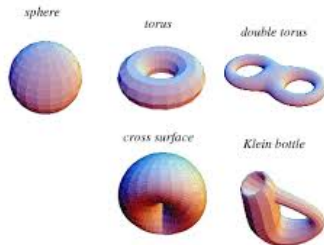**Some Definitions and Examples**
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Figure : Surfaces

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Figure : Surfaces

Why do we care?

- Surfaces are special 2-dimensional spaces.

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Why do we care?

- Surfaces are special 2-dimensional spaces.
- What is the shape of the universe?

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- Surfaces are special 2-dimensional spaces.
- What is the shape of the universe?
- Similar problem: we can only see a small piece of the universe at any point.

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Why do we care?

- Surfaces are special 2-dimensional spaces.
- What is the shape of the universe?
- Similar problem: we can only see a small piece of the universe at any point.
- How can we understand how these small pieces of 3-dimensional space patch together to build the universe? What is its overall shape?

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Why do we care?

- Surfaces are special 2-dimensional spaces.
- What is the shape of the universe?
- Similar problem: we can only see a small piece of the universe at any point.
- How can we understand how these small pieces of 3-dimensional space patch together to build the universe? What is its overall shape?
- Need to have mathematical theory that describes these shapes.

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Building surfaces

- As we saw, its hard to figure out how surfaces can be patched together out of flat pieces.

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- To visualize this "patching," we need good *building blocks*.

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Building surfaces

- As we saw, its hard to figure out how surfaces can be patched together out of flat pieces.
- To visualize this "patching," we need good *building blocks*.
- Disks:
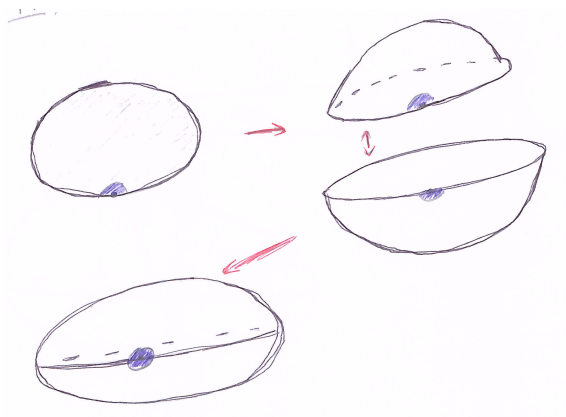

Figure : Disk

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Gluing Disks

Figure : Gluing Disks

Pants

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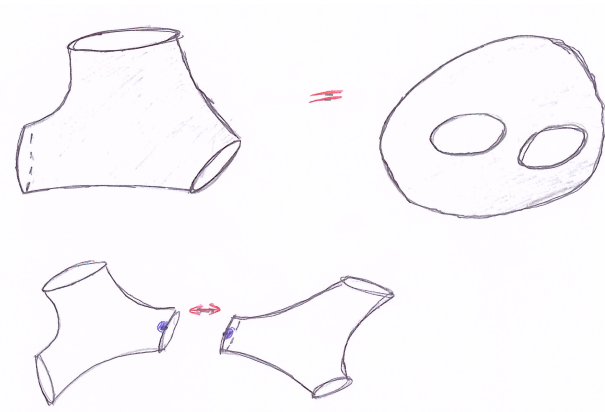
Pants

Figure : Pairs of Pants

Surfaces Built from 1 Pair of Pants

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Activity

What are ALL the surfaces that you can build with 2 pairs of pants and

- 4 disks
- 2 disks
- 0 disks

Pants Decompositions

- We will now build surfaces using ONLY pairs of pants.

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Pants Decompositions

- We will now build surfaces using ONLY pairs of pants.
- Problem: For any surface, there are many ways to build it using pairs of pants.

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Pants Decompositions

- We will now build surfaces using ONLY pairs of pants.
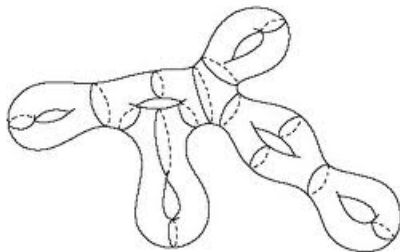- Problem: For any surface, there are many ways to build it using pairs of pants.
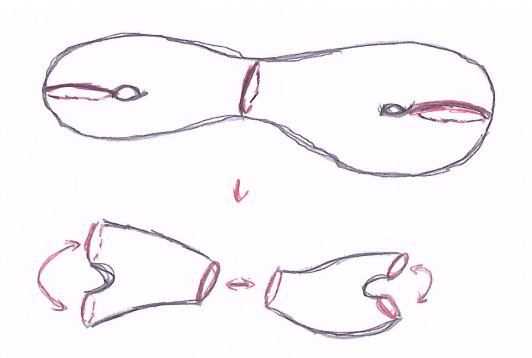

Figure : Pant Decompositions

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Example

Figure : Pants Decomposition

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Pants Moves

Figure : Pants Moves

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Using the Moves

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Activity

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Thank you.